Lagrangian Coherent Structures deduced from HF radar measurements

Francesco Enrile*, Giovanni Besio*, Marcello G. Magaldi†, Carlo Mantovani†, Simone Cosoli‡, Riccardo Gerin‡, Pierre Marie Poulain‡
*Università degli studi di Genova,
DICCA-Dipartimento di Ingegneria Civile, Chimica e Ambientale
Via Montallegro 1, 16145 Genova, Italy
e-mail: francesco.enrile@edu.unige.it
†ISMAR - Istituto di Scienze Marine
Forte Santa Teresa, 19032 Pozzuolo di Lerici (SP), Italy
‡OGS - Istituto Nazionale di Oceanografia e Geofisica Sperimentale,
Borgo Grotta Gigante 42/C, 34010 Sgonico (TS), Italy

Abstract—**In the present paper Lagrangian Coherent Structures are detected in the Gulf of Trieste, i.e., a Gulf located in the north-eastern part of the Adriatic Sea. Lagrangian Coherent Structures are usually detected through Lyapunov exponent diagnostic tools. However, such diagnostics lack of a rigorous mathematical background and try to associate well-known classical dynamical system structures of autonomous dynamical systems to the features of fluid flows. Such flows are studied under the perspective of non-autonomous dynamical systems, neglecting diffusion. In this work we try to detect Lagrangian Coherent Structures, i.e., the key material lines that shape trajectory patterns, with both the established Finite-Time Lyapunov Exponents and the recently introduced rigorous mathematical definitions implemented in the publicly available MATLAB LCS Tool. A comparison between real and simulated drifter trajectories is considered, too.**

I. Introduction

Transport and mixing problems are of fundamental importance in several disciplines. In water bodies such phenomena have strong effects on the quality of water due to transport of pollutants. Two distinct processes govern the physics on hand: advection and diffusion. Diffusion usually develops over a time scale longer than the advection. Thanks to this reason, in the initial stages of mixing processes, it is possible to neglect diffusion and study transport phenomena on the basis of advection alone. Therefore, fluid particle trajectories are solution of ordinary differential equations:

$$\dot{\boldsymbol{x}} = \boldsymbol{v}\left(\boldsymbol{x}, t\right) \tag{1}$$

where the left hand side is the derivative with respect to time and the right hand side is the velocity of the fluid.

The resulting pattern of advection can be studied through the analysis of the corresponding non-autonomous dynamical system (i.e., time-dependent) described by equation (1). Classic dynamical system theory of autonomous systems (i.e., time-independent) reveals a wealth of structures influencing tracer trajectories. In autonomous dynamical systems fixed points and stable and unstable manifolds gain a decisive role in the development of fluid-particle trajectories [1].

In case of real fluid flows described by non-autonomous dynamical systems one must take into account not only the explicit dependence from time but also the finite nature of the phenomena. Such considerations led to the search for the analogous of stable and unstable manifolds that behave as transport barriers [2]. From this perspective the concept of Lagrangian Coherent Structures aroused as the most influential material line that shape trajectory patterns [3][4].

Heuristic indicators have been extensively used in order to detect these structures. Their identification relies mainly on the use of Lyapunov-exponent-based diagnostic tools, namely by locating ridges (i.e., local maxima), in Finite-Time Lyapunov Exponent (FTLE) scalar fields. FTLE applications are largely diffused, despite the fact that the current techniques employed in the literature can identify unequivocally actual LCSs only under more restrictive conditions [5][6]. The continuous recent use of the current techniques is supported by the fact that Lyapunov Exponents still represent a relatively simple and powerful mean to mark transport barriers and detect the directions along which transport is likely to develop [7]–[12].

A rigorous mathematical approach to this subject has been recently developed by [5][6], providing a theoretical background that could be able to overcome the present inconsistencies of the heuristic approach. Such material lines should distinguish themselves by attracting or repelling nearby trajectories at the highest rate in the flow.

The present work examines the differences between Finite-Time Lyapunov Exponent scalar fields and Lagrangian Coherent Structures detected according to [5][6][13] in the Gulf of Trieste (GoT), located in the north-eastern Adriatic Sea, see Figure 1. The identified structures will be compared with real drifter trajectories deployed during the TOSCA (Tracking Oil Spills & Coastal Awareness network) project field campaign.

II. TOSCA project

TOSCA (Tracking Oil Spills & Coastal Awareness network) project aims at improving the quality and effectiveness of decision-making in case of marine accidents concerning oil spills and search and rescue operations (SAR) in the Mediterranean Sea. The project aims at providing real-time observations and forecasts of marine environmental conditions

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in the Western and Eastern part of the Mediterranean Sea. Gathered data are combined in a useful decision support tool for authorities in charge of marine emergency response. In particular, in the GoT, which could be defined as the region of the Adriatic Sea north-east of the ideal line connecting Savudrija and Grado, a network of three CODAR (COastal raDAR) was installed in order to measure surface velocity fields. The three radars are located in Aurisina, Barcola and Pirano spots (see Figure 1). The velocity data were collected between the 23^{rd} and 30^{th} of April 2012 with a spatial resolution of 1.5 km and a time resolution of 1 hour. In the field campaign drifters were deployed, too. The trajectories of the drifters deployed in the sea were recorded thanks to GPS devices. Their positions were subsampled via kriging-interpolation methods to work with the same time resolution of the velocity fields.

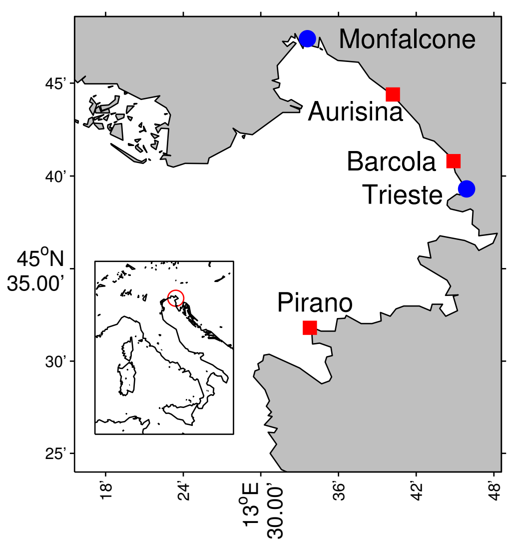

Fig. 1. Radar network locations in the Gulf of Trieste.

III. Finite-Time Lyapunov Exponents

The detection of LCSs by FTLEs is pursued according to [4]. In this context FTLEs can be considered a finite-time average of the maximum expansion rate that a couple of particles advected by the flow can experience in a finite-time interval T, called integration time. The definition of the FTLE reads

$$\sigma_{t_0}^{t_0+T}(\boldsymbol{x}) = \frac{1}{|T|} \log \sqrt{(\lambda_{max})} \tag{2}$$

where λ_{max} is the maximum eigenvalue of the Cauchy-Green tensor, t_0 is the initial time and T is the integration time, i.e., the finite-time interval over which the FTLE is calculated. Defining the deformation gradient as

$$\boldsymbol{F} = \frac{d\boldsymbol{x}(t_0 + T)}{d\boldsymbol{x}(t_0)} \tag{3}$$

the Cauchy-Green Tensor is evaluated as

$$\boldsymbol{C_G} = \boldsymbol{F}^T \boldsymbol{F} \quad . \tag{4}$$

The Cauchy-Green tensor is a linear operator represented by a symmetric and positive definite matrix that expresses a rotation-independent measure of deformation, since a pure rotation does not produce any strain [14]. The FTLE values are computed through a finite-difference scheme [15] over a regular grid. The values associated with the nodes of the grid form a scalar field. In [4], Lagrangian Coherent Structures are defined as the ridges of Finite-Time Lyapunov Exponent fields.

Despite several recent issues aroused around the evaluation of the flux across the ridges of FTLE fields [1] [5], a property has generally been associated to these structures: the flux across them is very small and if they are actual Lagrangian Coherent Structures the flux is null.

It is worthwhile to note that FTLEs operate with a fixed time-scale T and detect a separation rate that changes from point to point. Detection of Lagrangian barriers leads to the detection of two different types of structures that behave in opposite ways: repelling and attractive structures are commonly presented in literature [4][16][7]. These features are calculated with forward and backward particle trajectory integration in time, respectively.

IV. Hyperbolic Lagrangian Coherent Structures

Recent works seek Lagrangian Coherent Structures on the basis of a rigorous mathematical formulation [1] [5] [17] [6]. Over the time interval of interest the Authors define Lagrangian Coherent Structures as the prevailing attracting or repelling material lines. Considering a material line at the initial time, a unit normal vector $\boldsymbol{n}_0(\boldsymbol{x}_0)$ to this line in the point $\boldsymbol{x}_0$ will change orientation over the time interval of interest and generally will not remain normal to the material line. Recalling that a generic vector $\boldsymbol{\xi}$ at the initial time evolves, under the linearized flow, into $\boldsymbol{F\xi}$ and by defining the repulsion rate $\rho(\boldsymbol{x}_0, \boldsymbol{n}_0) = \langle \boldsymbol{n}_t, \boldsymbol{F}\boldsymbol{n}_0 \rangle$ as the scalar product between the evolved initial unit vector $\boldsymbol{F}\boldsymbol{n}_0$ and a new unit vector $\boldsymbol{n}_t$ normal to the advected material line, it is possible to evaluate the behaviour of the material line over the time interval of interest. If the repulsion rate ρ is greater than 1, the material line will exert net normal repulsion on nearby fluid elements. On the contrary, it will exert along its normal direction attraction over the nearby fluid elements. [17] find that the initial positions of Hyperbolic LCSs must be orthogonal to a specific vector field. For two dimensional flows repelling LCSs must be trajectories of the differential equation

$$\boldsymbol{r}' = \boldsymbol{\xi}_1(\boldsymbol{r}) \tag{5}$$

and attracting LCSs must be trajectories of the differential equation

$$\boldsymbol{r}' = \boldsymbol{\xi}_2(\boldsymbol{r}) \tag{6}$$

where $\boldsymbol{\xi}_1$ and $\boldsymbol{\xi}_2$ are the eigenvectors of the Cauchy-Green tensor associated with the minor and the maximum eigenvalues, respectively. The numerical detection of these LCSs is carried out by the publicly available software LCS Tool [13].

V. Lagrangian Coherent Structures in the Gulf of Trieste

Thanks to the available velocity fields of the sea surface of the Gulf of Trieste, a comparison between Finite-Time Lyapunov Exponents fields and Lagrangian Coherent Structures detected by LCS Tool [13] is carried out. A further investigation is developed since real drifter trajectories are available and a direct comparison between drifters data and LCS is possible.

A. FTLE fields and Lagrangian Coherent Structures

[4] associate ridges of Finite-Time Lyapunov Exponents fields carried out with a forward integration with repelling structures. In Figure 2 the underlying field is the FTLE forward evaluated with an integration time T of 25 hours. The thin blue lines represent the hyperbolic repelling LCSs computed over the same time interval calculated via the LCS Tool [13]. It is possible to see that the ridges of the FTLE field and the LCSs do not perfectly superimpose. A good agreement between the FTLE pattern and LCSs is present especially comparing the structure located north-west of the GoT that closes the Gulf of Monfalcone, i.e., an internal Gulf of the GoT.

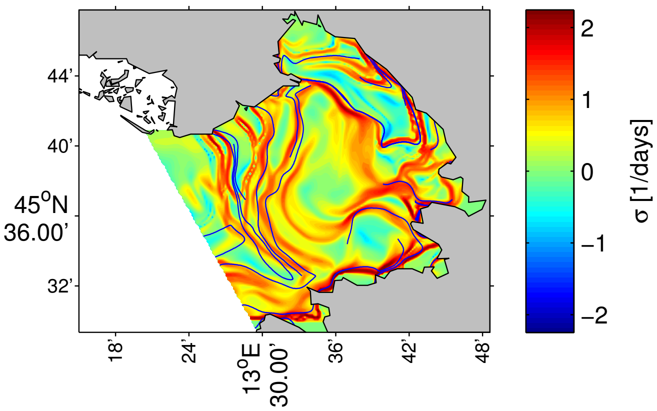

Fig. 2. Forward Finite-Time Lyapunov Exponent field for 00 UTC of the 23^{rd} of April 2012, and repelling LCSs (thin blue lines).

[18] show that ridges of FTLE coincide in many cases well with material structures. However, in general, they are not exact material structures, no matter which ridge definition of a scalar field is adopted. Unless very long integration times are used, FTLE ridges can deviate considerably from material structures.

Comparisons like those presented in Figure 2 could be further investigated numerically, evaluating the flux across the ridges of the scalar field and the flux across the LCSs. However, such evaluations are carried out on the basis of velocity fields whose reliability is taken for granted. Whether or not ridges of FTLE fields and LCSs represent with good approximations material boundaries, the most important basic question remains: how reliable are the measured velocity fields? To address this question, in the following we compare trajectories obtained by integrating the measured velocity fields and the Lagrangian real observed trajectories.

B. Drifters in the Gulf of Trieste

During the TOSCA project drifters were deployed in the GoT making possible the comparison among drifter trajectories and Lagrangian structures. Taking into account the trajectory of a real drifter known with an hourly time step, it is possible to compare such drifter trajectory and the LCS obtained with the approaches described in Section V-A. Two types of simulations

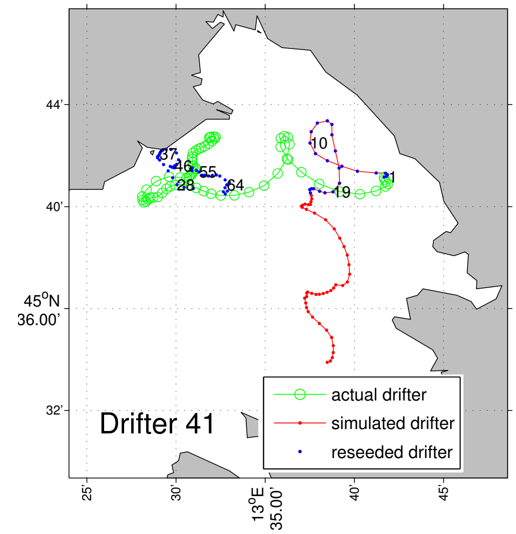

Fig. 3. Trajectories of actual drifters in green, simulated in red and reseeded in blue. The numbers show the evolution of the reseed drifter (blue). The starting point is common to all three.

are carried out. In the first type the simulated drifter is re-initialized every 24 hours on the coordinates of the observed real drifter (24-hour reseeding procedure). In the second type, the simulated trajectory evolves freely for the whole duration of the simulation and no reseeding procedure is applied. We focus the analysis of the results of the simulation on one of the deployed TOSCA drifters, namely drifter 41. The real and simulated trajectories for this drifter are shown in Figure 3. The real drifter is represented in green, the simulated drifter in red and the simulated reseeded drifter in blue. In Figure 3, the numbers alongside the reseeded drifter mark the evolution of the trajectory: every number is associated with a different time step. The starting point for all the three drifters is the same and identified by the number 1. The real and simulated drifters diverge, i.e., the trajectories are quite different because the real and the simulated drifters tend to move at the opposite sides of the GoT. In Figure 4 some snapshots of the evolution of the drifters superimposed to the FTLE fields and LCSs are depicted. On the left Panels the entire FTLE field is depicted, while on the right Panels only the ridges (in red) are shown. Blue lines are Hyperbolic LCSs computed by the MATLAB LCS Tool [13].

From Panels g) and h) it is possible to see that the final evolution of the simulated and real drifters over the time interval of interest ends at the opposite sides of the GoT. This difference in trajectories is well known [19] and due to, for example, high sensitivity to initial conditions and coarseness of the velocity field. Concentrating on FTLE fields, the reader would argue from panels a) and b) that the drifter would move preferentially along the north-south direction, being the drifter on the left of a FTLE ridge pointing south. Actually, it would move in the east-west direction in the next time steps. Panels a) and b) depict the moment of deployment, when the positions of real and simulated drifters are the same. This behaviour is readable from FTLEs ridges only from panels e) and f) whereas actual Hyperbolic LCSs evaluated according to [13] had already shown this possible evolution from panels a) and b).

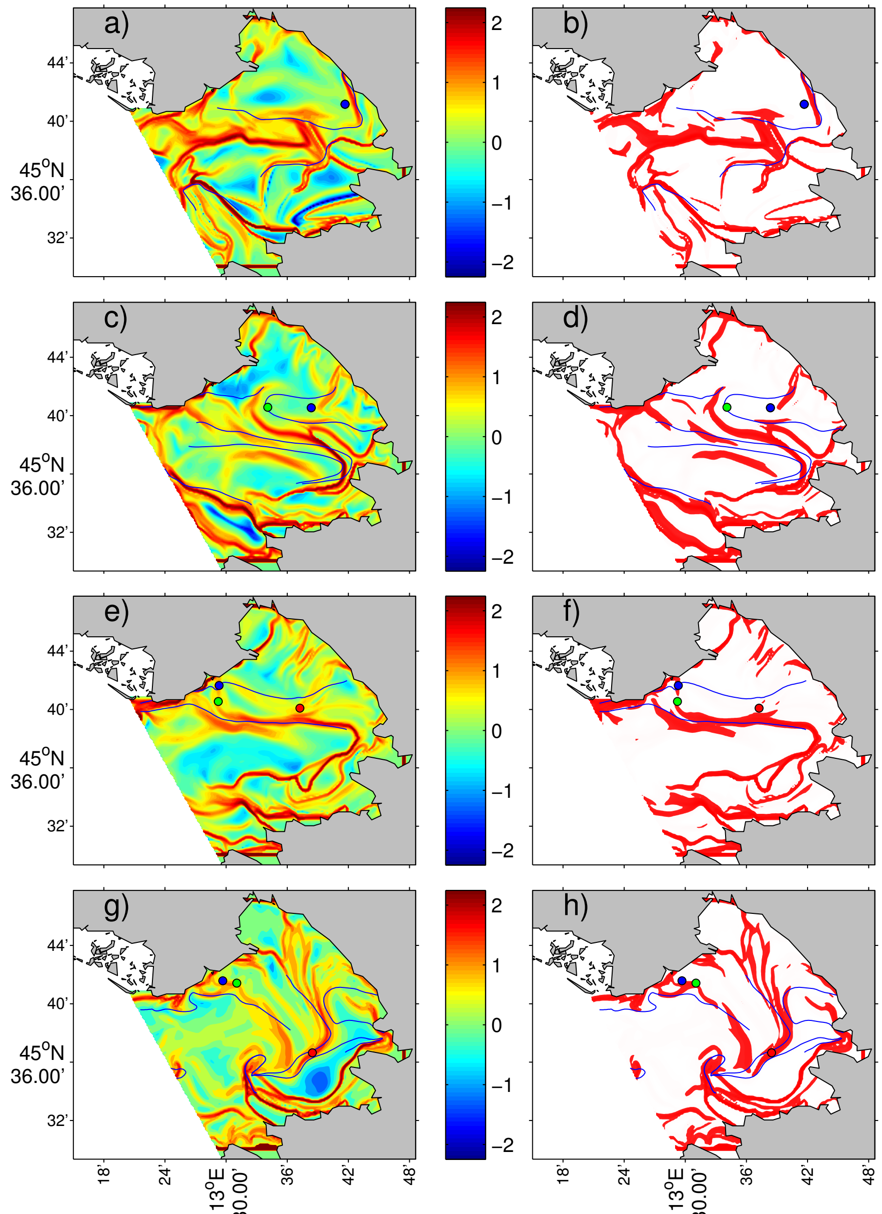

Fig. 4. Backward Finite-Time Lyapunov Exponent fields and attractive LCSs in blue. Real drifter in green, simulated drifter in red and simulated reseeded drifter in blue. Left Panels show the whole FTLE field and hyperpolic LCSs (thin blue lines). Right Panels show only FTLE ridges (in red) and hyperpolic LCSs (thin blue lines).

These findings suggest then that FTLE ridges can give reasonable information about the most probable direction of spreading of passive tracers even if they are not able to reproduce accurately the information provided by LCS Tool.

VI. Conclusion

In this work we detected Lagrangian Coherent Structures from a heuristic indicator such as Finite-Time Lyapunov Exponents and from a rigorous mathematical definition in the Gulf of Trieste. The input velocity fields are those measured by the network of coastal radars of the TOSCA project. Lagrangian structures are subsequently compared with field measurements of drifter motion. A particularly interesting drifter is taken into consideration and two different kinematic simulations are carried out: one with a daily reseeding of the simulated drifter on the coordinates of the real one and another without reseeding. The trajectories are analysed at the light of the identified Lagrangian Coherent Structures. Such an analysis shows how useful Lagrangian structures are in studying drifter motion and underlines that the new definitions of LCSs introduced by [5] [6] could be important in finding true material lines that shape trajectory patterns of passive tracers. However, further analysis based on field data are to be carried out in order to assess the reliability of LCSs to provide correct information about the transport of mass in geophysical flows. Besides, further investigations are needed to assess reliability of the measured velocity fields.

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