

Simplified Calculation of Constituent Tidal Currents and Height from HF Radar Profiles across the Mouth of Bays and Sounds

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Abstract— A dominant part of the circulation in nearly enclosed bays, estuaries, or sounds is dictated by tidal inflow at its mouth, called co-oscillatory forcing. The remaining flow component is usually due to winds. HF radar measurements over an area at the entrance can be used to determine the sinusoidal tidal velocity constituents along a line across the mouth. We use this complex spatial profile at different phases of the tidal cycle as the boundary excitation condition to solve a scalar second-order partial differential equation (PDE) for tide height. For the remaining boundary condition, the flow normal to the shore is taken to be zero. The bathymetry of the bay is included in the PDE. This is then solved by a powerful finite-element code, PDE2D. From the tide height distribution, the velocity circulation is simply calculated as its gradient.

We present results applying this method to simple, canonical bay shapes and bathymetries. The effect of the bottom shape is studied, as well as the different excitation profiles at the mouth. Both tide height and vector current field are calculated and compared for the different geometries and excitations. Our future studies are applying this to Long Island Sound, for example, where three SeaSondes straddle the mouth at the Eastern end, owned and operated by University of Rhode Island and University of Connecticut. Our studies reported here of canonical bay and bottom shapes serves as a guide and check in applications to these real-world situations.

To our present lowest order of approximation, friction and dissipative effects that cause tidal-phase time lags at different points are neglected. Higher-order nonlinearities are also neglected. Both of these effects are being included in subsequent studies. The main advantage of our method for co-oscillatory tidal analysis is simplicity: it avoids the complexities and computational requirements of full-up numerical primitive equation methods. The goal is to provide this as an algorithmic tool to run on the standard PCs that control and process data for the many HF radars being operated in bays. From a radar-measured profile of the tidal constituents across the mouth, we hope to estimate tidal circulation and tide heights throughout

the bay, in areas well away from the entrance where HF radars make their measurements.

I. METHODOLOGY

In recent years there has been a dramatic increase in the capability of observations of estuarine and coastal regions. This is due largely to the proliferation of HF radars [1], [2]. Furthermore, with the advent of fast computers, oceanographers have both the observational and computational capabilities unimagined years ago to perform tidal studies. It is known that tidal currents in nearly enclosed bays are dominated by the tidal inflow at their mouth. Therefore, in our work we use HF radar measurements over an area at the entrance to determine the sinusoidal tidal velocity constituents along a line across the mouth. This complex spatial profile at different phases of the tidal cycle is used as the boundary excitation condition to solve a scalar second-order partial differential equation for tide height, and from this, the tidal currents everywhere.

In this research work we analyze in particular, the effect of depth on tides in bays. Currently, there are full primitive equation models that produce tide heights in a comprehensive way. However, these models are computationally very time intensive and somewhat cumbersome. We present a model that is simple, yet very efficient to study the dependence on depth of tide heights in bays. This method, when used in conjunction with HF radar data, can provide accurate results for the velocity profiles in bays.

In our methodology, we use the equation of motion (to lowest order):

$$= -\frac{1}{g} \frac{\mathbf{V}}{t} = -\frac{j}{g} \mathbf{V}$$

where (\mathbf{x}, t) is the surface height above a mean level as a function of horizontal position, \mathbf{x} and time t , and \mathbf{V} is the velocity. Bold face letters denote 2-D vectors. The j replaces differentiation with respect to time when variation is sinusoidal, as it is for tidal constituents, and removes explicit time dependence from height and velocity.

$$\nabla \cdot (h\mathbf{V}) = -\frac{\eta}{t} = -j\eta$$

The equation for conservation of fluid to lowest order is: where $h(\mathbf{x})$ is the water depth below the mean height level. Combining these two equations, one obtains the following PDEs (partial differential equations):

$$\nabla \cdot (h\mathbf{V}) + \frac{\partial^2 \eta}{\partial t^2} = 0$$

$$\nabla \cdot (h\mathbf{V}) + \frac{\partial^2 \eta}{\partial t^2} = 0$$

Using the boundary conditions, e.g., that the normal flow into the coastline is zero and that the normal flow across the mouth comes from SeaSonde profile tidal analysis, the scalar Helmholtz equation for tide height, $\eta(\mathbf{x})$, is solved using the PDE2D finite-element software [3].

II. RESULTS

We present here results for two canonical bay shapes: semi-elliptical and rectangular. Each is $x = 100$ km long and $y = 40$ km at its widest in the elliptical case. These allow us to verify the correctness of our results against analytical solutions. Furthermore, studying canonical shapes has provided us with greater insight into the physics of the phenomena, since many bays resemble roughly these shapes. Subsequent work will apply this validated methodology to bays across the United States.

In Figs. 1 and 2 we present tide height, $\eta(\mathbf{x})$, for rectangular and elliptical bays with a constant depth of 100 m. The remaining figures show velocity flows. Figs. 3 and 4 refer to a constant depth case and Figs. 5 and 6 refer to a variable depth case. Finally, Fig. 7 shows a comparison between a constant and a variable case for a maximum center depth of 25 m. All of the results presented here had a sinusoidal inflow profile at the inlet, being zero at the mouth center. Figs. 4 and 6 show a negligible difference when changing the maximum depth of $h(\mathbf{x})$ between 25 and 100 m.

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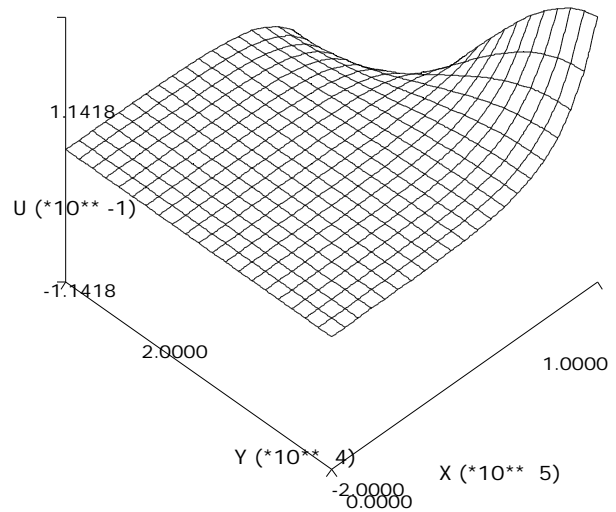


Figure 1. $\eta(\mathbf{x})$ for a rectangular shape with constant depth $h = 100$ m

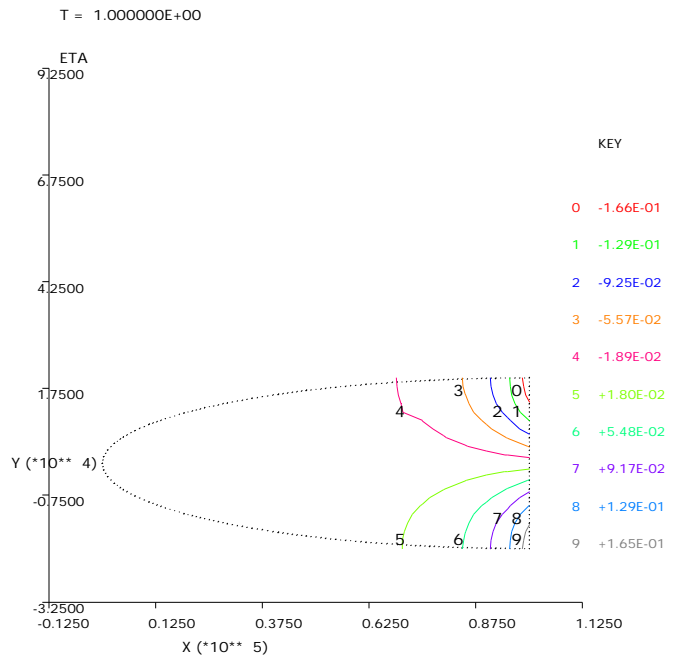


Figure 2. $\eta(\mathbf{x})$ for an elliptical shape with constant depth $h = 100$ m

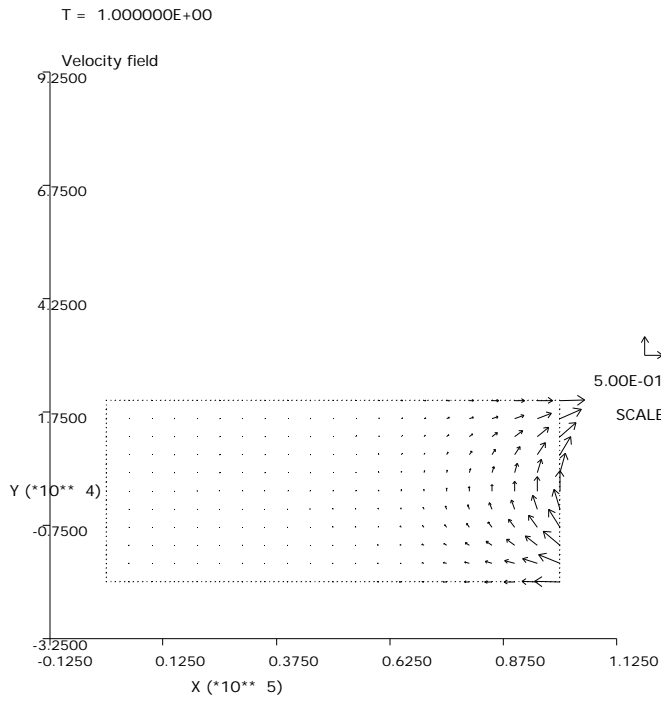


Figure 3. Velocity profile for constant depth $h = 25$ m

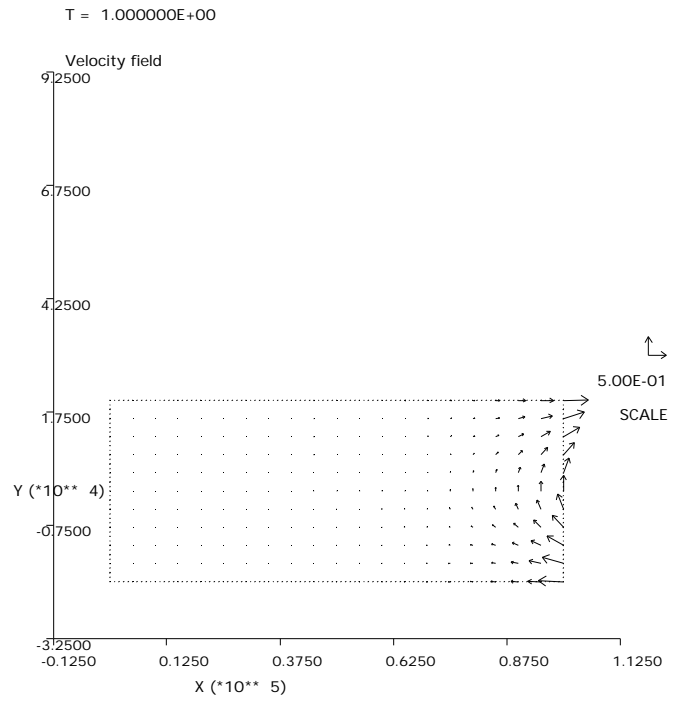


Figure 5. Velocity profiles for variable h , maximum depth at center being 25 m.

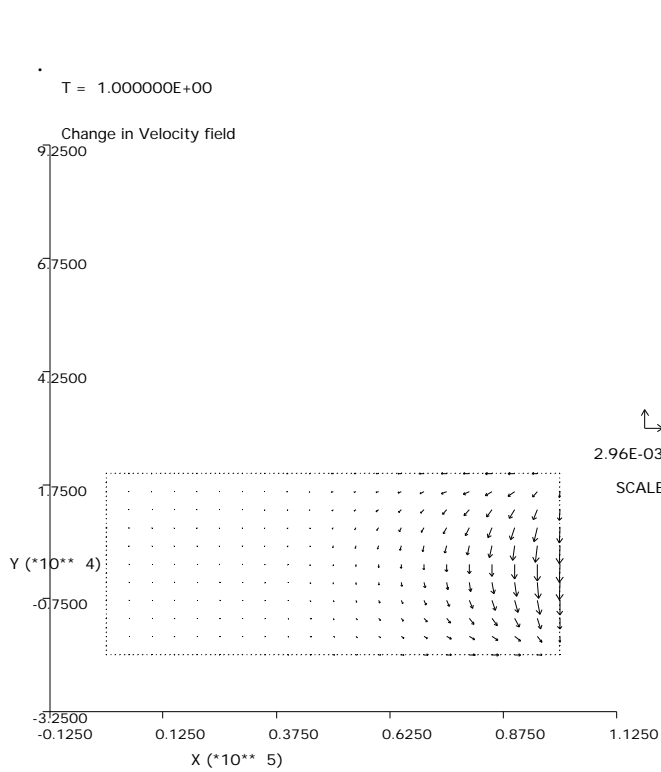


Figure 4 Difference between velocity profiles for constant depth values of h for 25 and 100 m.

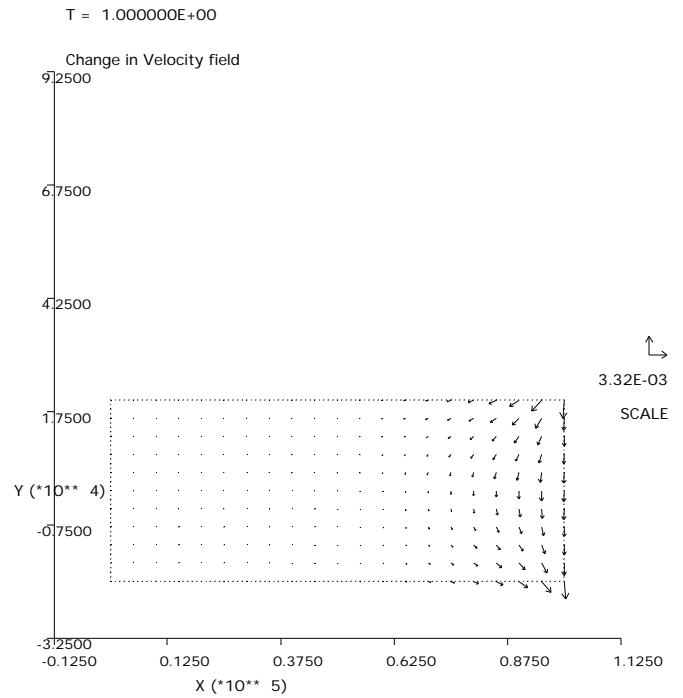


Figure 6 Difference between velocity profiles for variable h , being 25 and 100 m maximum depths at the center

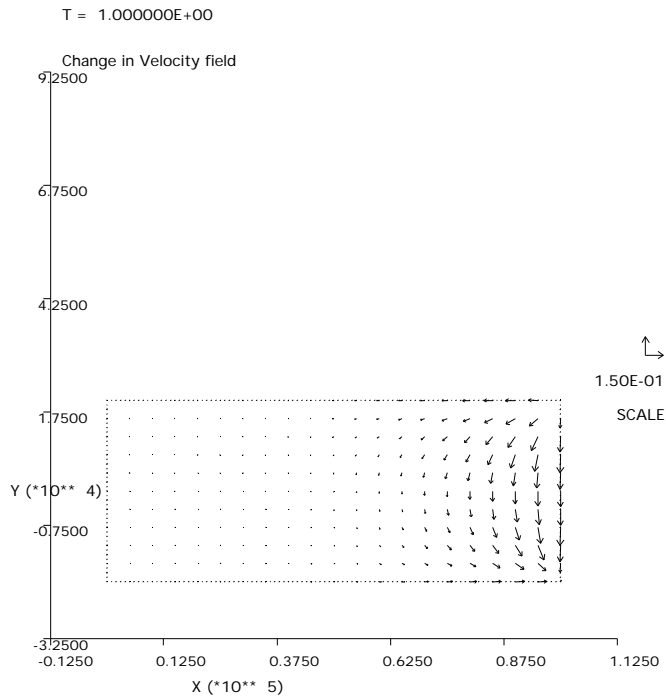


Figure 7 Difference between velocity profiles for variable and constant $h(\mathbf{x})$, with the maximum depth at the centers being 25 m.

III. CONCLUSIONS

After analysis of numerous computer runs, we conclude that whether $h(\mathbf{x})$ is constant or a variable, increasing H_{\max} beyond a certain limit has little effect on tide height $\eta(\mathbf{x})$ and velocity $\mathbf{V}(\mathbf{x})$. This is because the second Helmholtz term in our PDEs becomes negligible as the first term increases with $h(\mathbf{x})$. In addition, it is found that the velocity profiles depend on whether a constant or a variable bay bottom depth is used. Qualitatively these results are in accordance with observations of actual bays.

Future work will use this methodology on selected bays around the U.S., making use of data acquired with HF radars.

IV. REFERENCES

1. Lipphart, B. L., Jr., A.D. Kirwan, C.E. Grosch, J.K. Lewis and J.D. Paduan, Blending HF radar and model velocities in Monterey Bay through normal mode analysis, *Journal of Geophysical Research*, vol. 105, 3425-3450, 2000.
2. Kohut, J., Glenn, S., and D. E. Barrick, SeaSonde is integral to coastal flow model development, *Hydro International*, vol. 3, 32-35, 1998.
3. Sewell, G., PDE2D: Easy-to-use Software for General Two-Dimensional Partial Differential Equations, *Advances in Engineering Software*, vol. 17, 105-112, 1993.