When Are HF-Radar Observed Wave Heights Modulated by Periodic Tidal and Inertial Currents?

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***Abstract*—Two recent sets of HF radar observations in the Atlantic by SeaSondes have observed unexpected periodic modulations of significant wave height at the M2 tidal and inertial frequencies. These are examined and related to the underlying, strong currents. They are interpreted in terms of kinematic advection influences that change the first-order wave characteristics due to frequency dispersion.**

Keywords—HF radar; wave monitoring; inertial oscillations; tidal modulation of wave height

I. INTRODUCTION

Coastal HF radars have been used to map ocean surface currents and monitor wave fields over four decades [1, 2, 3]. Currents are observed from the Doppler shifts of the dominant Bragg-scattering surface gravity waves. Sea state is extracted from the weaker second-order spectral echoes surrounding the Bragg peaks, produced by electromagnetic and hydrodynamic interactions among the surface waves. The type of radar under discussion here is the CODAR SeaSonde®, which has a compact antenna system and is based on direction-finding (DF) principles to determine echo bearing [3, 4].

Operational wave parametera (e.g., significant waveheight) are usually outputted hourly. Surface wavefields are normally considered to be independent phenomena from ocean currents. Therefore, one normally does not expect to see periodic oscillations in them that correlate with those found in currents, such as tides and inertial oscillations. Masson [5] observed by buoy wave height variations at M2 semi-diurnal tidal periods that varied with current strength. These variations depended on water depth at nearby buoy locations. These were related to different current strengths at the two buoy locations, as mapped by SeaSonde HF radars. She analyzed a convective mechanism to explain this periodic modulation of waves by tides that was advanced by Huang et al. [6]. When tidal current strengthened as it flowed over a shallower shelf, the periodic wave height modulation increased.

This wave height modulation by semi-diurnal tides over a shallow seamount was recently observed for our HF radar outputs in a region North of Scotland between the Orkney and Shetland Islands. Two ~5 MHz SeaSondes straddled this region, as shown in Fig. 1.

In this presentation and manuscript, we focus on further analysis of this North Sea data set, to confirm conclusively this M2 tidal cause; we examine the spectrum of the time series of this modulation in the next section. Then we discuss and confirm the existence of wave parameter modulations by inertial oscillations in currents on the West coast of Spain, also operated near 5 MHz. Finally, we discuss the mechanism put forward by Huang et al. in [6] that explains the modulation of wave height by periodic current fluctuations.

II. SEMI-DIURNAL WAVE MODULATIONS AT BRAHAN

A. Region of the Measurements

Fig. 1 shows the North Sea area where SeaSonde 5 MHz current and wave measurements were made during 2013-2014. The baseline between the radar sites 84 km apart (shown at NRON and SUMB) runs SW to NE. Tidal currents flow primarily perpendicular to this baseline, dominated by the M2 semi-diurnal component. Models and measurements show that the tidal flow strengthens as it flows over the shallower shelf between these island groups, being weaker in deep water to the NW where a U.K. met buoy is located that provides wave, current, and wind data. The campaign was named the Brahan

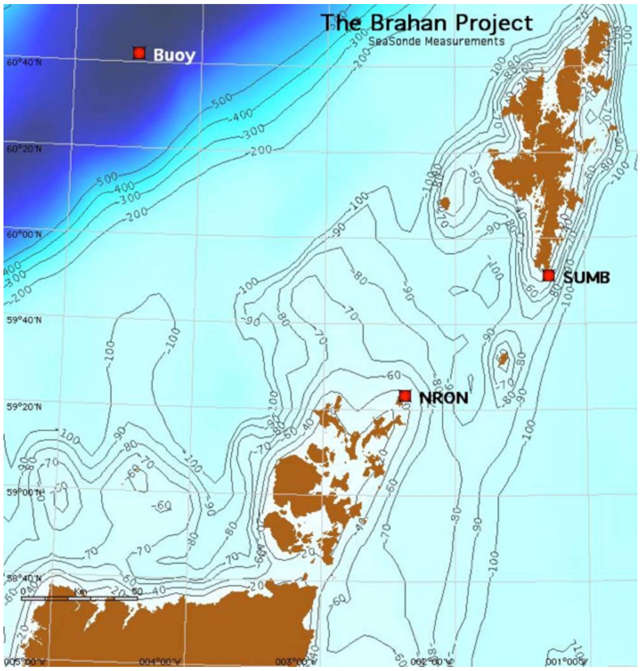

Figure 1. Chart showing area of SeaSonde 5 MHz measurements, with radar sites at NRON (Orkney) and SUMB (Shetland) Islands. U.K. Met buoy is located at red circle marker to the Northwest

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project, after a local legendary seer of the 17th century. See [7] for more details of these Brahan measurements.

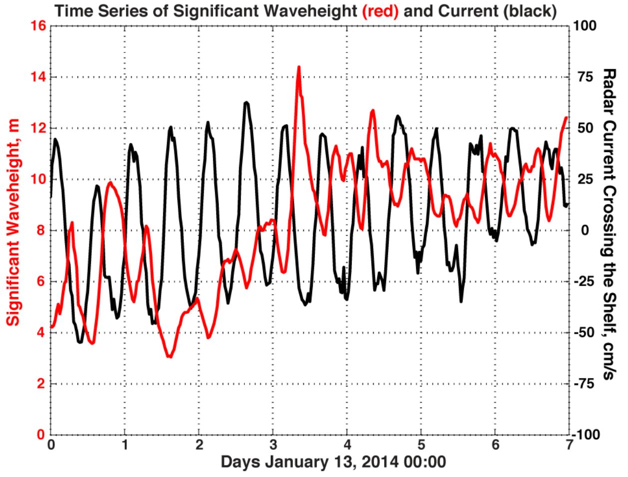

Figure 2. Plot of significant wave height (red) and cross-shelf current (black) over one week period with significant wave periodicity.

B. *Example of Radar Observed Wave Height and Current*

We picked a one-week period to show in the oscillations at the M2 semi-diurnal tidal frequency in Fig. 2. The red curve is the significant wave height. This is an average over ten semi-circular range cells from each radar. We found that significant wave height varied little over this spatial region, justfying our average to reduce noisiness. As is evident, this was a period of high seas. We remark that the wave height at the buoy 240 km to the NW in very deep water had none of the oscillations seen over the shelf in the radar measurements, although the average through these oscillations was the same level with a several hour delay. The tidal current was much weaker at the buoy.

It was a period of strong currents across the radar baseline, dominated by the semi-diurnal M2. This was calculated as the crossing flow component at the midpoint between the two sites. Because the radars cannot measure this crossing component directly (they measure only the radial flow), we used an interpolation procedure that has been demonstrated to be successful, stretching between two points on either side where total vectors are accurately estimated (e.g., see [7]).

Although semi-diurnal fluctuations are unmistakeable in the current, they show up in signficant wave height also. The period is the same; this was confirmed from temporal spectra done from these time series, that are shown in the presentation. Temporal oscillations were also seen in the wave period and direction plots, as well as Bragg-wave histories that are used to get wind direction [7].

Another interesting fact is confirmed in these comparison plots. The periodic oscillations are nearly 180° out of phase with each other between currents and waves. This was predicted in the theoretical work of Huang et al. [6] and experimental observations by Masson [5], where wave height was measured with a wave buoy. In other words, the modulation becomes stronger when the current and wave directions oppose each other. During this week, this nearly opposing condition was confirmed [7]. The "opposing" colinear phase condition, as measured from our data, is actually about 150°, i.e., currents and waves were not always precisely opposite. The percent height modulation was about 14%, similar to that predicted [7] and found by Masson [5].

III. INERTIAL OSCILLATIONS AT SILLEIRO, SPAIN

A. *Region of the Measurements*

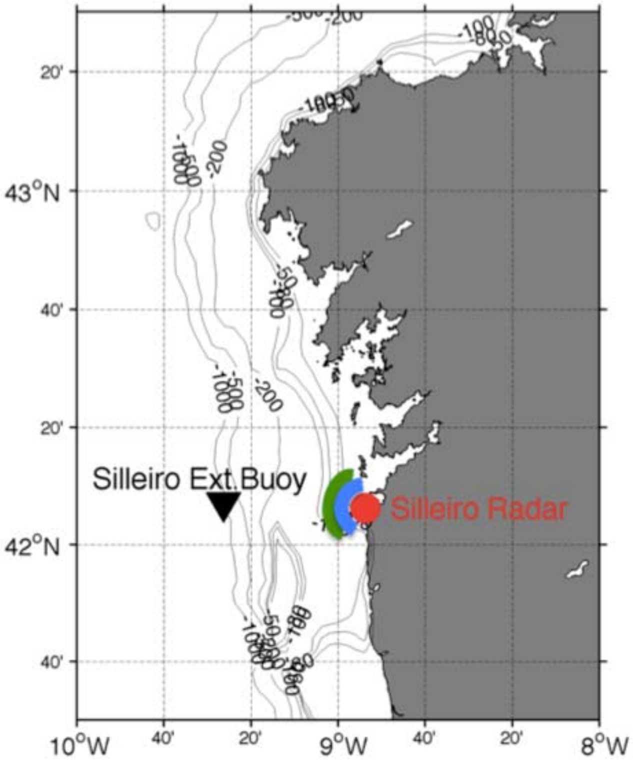

Figure 3. Chart shows region and bathymetry of 5 MHz SeaSonde radar at Silleiro, Spain. Depth contours are meters. External buoy is 45 km from shore.

Fig. 3 shows the West coast of Spain, near the Portuguese border, where one of several 5 MHz SeaSondes has been operating as part of the network of Puertos del Estado. Exposed to storms from the North Atlantic, high waves and that generate inertial current responses are often seen. We focus on a period during fall and winter during 2014. The radar range resolution cell size is 5.85 km. A buoy with wave, current, and wind sensors is located 45 km directly offshore from the radar.

B. *Examples of Radar Wave and Buoy Current Observations*

We select an eight-day period that exhibited significant periodic oscillations in wave height and other wave parameters. The data were averaged over about 2.5 hours to suppress high-frequency noisiness. Wave data from SeaSondes are measured as averages over each semi-circular range cell [7]. Range Cells 2 and 3 are shown in blue and green in Fig. 3. For these studies, wave parameters from four range cells from ~11 km to ~33 km were averaged together, after it was determined that wave parameters did not vary within this range span. Although depths spanned 35 m to 400 m within these ranges, predominant onshore waves from the NW ensured a homogeneous wave field. At the buoy, the depth was ~1000 m.

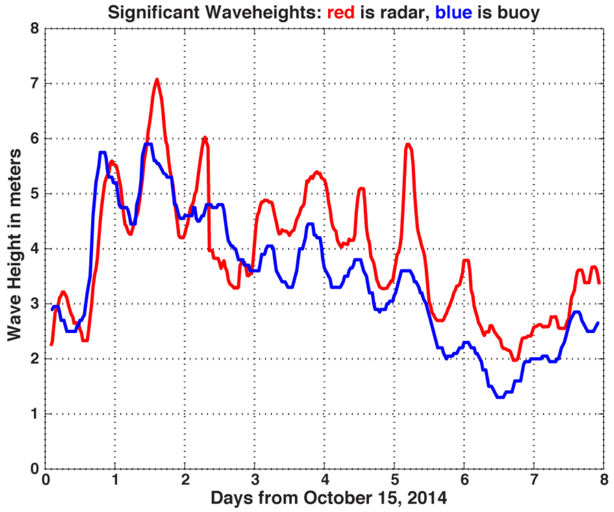

Figure 4. History over eight days of significant wave height from the radar (red) and buoy vertical accelerometer (blue).

Fig. 4 above shows the significant wave heights from the radar and the buoy over an eight-day stormy period. Although not plotted here, the periods ranged between 13 and 16 s. The inertial period for the latitude here of about 42° is 17.9 hours, determined from the equation:

$$\Omega_{\text{Inertial}} = 2\Omega_{\text{Earth}} \sin(\text{Latitude}) \quad (1)$$

where Ω is frequency (in the same units on both sides of the equation), relating earth rotation frequency to inertial frequency.

Spectral confirmation from the above time series shows that indeed, the dominant oscillations seen by both sensors fall exactly at this period; we discuss these in our presentation.

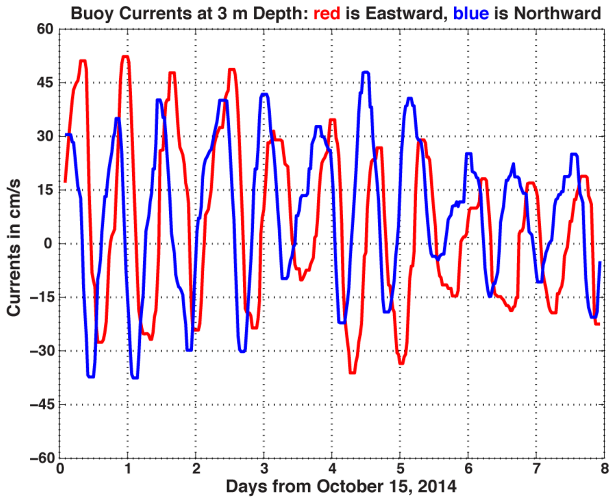

Figure 4. Eastward (red) and Northward (blue) components of current measured at buoy 3 m below surface.

Fig. 4 shows the currents at the buoy, 45 m from the coast measured 3 m below the free surface. The fluctuations dominating for this period are inertial oscillations. This is confirmed by the fact that the Northward component leads the Eastward by a quarter cycle, expected from the clockwise-rotating features in the Northern hemisphere induced by the wind gustiness of the storms dominating the area.

Wind speed during this period exceeded 20 m/s except toward the end. The fact that both plots above show a gradual reduction in the wind speed, in the oscillations, and in the wave heights presents a convincing case for the inertial-current explanation of the periodic wave fluctuations.

IV. MECHANISM FOR WAVE-CURRENT PERIODIC COUPLING

Several papers have discussed hydrodynamic modulations of waves by underlying currents. However, the explanation that best fits the measurements here as well as those of Masson [5] were put forward by Huang [6].

This relatively simple theory is linear. Its basis is the advective (or Doppler) effect of the two fields moving at different velocities. The radian frequency, ω, at which a fixed sensor will observe a wave riding on top of a current is given by:

$$\omega = kV_W + \sigma \quad (2)$$

where k is the wavenumber of the surface gravity wave in the direction of the current; V_C is the velocity of the current; and σ is the intrinsic radian frequency of the surface wave under the influence of gravity, g, is given in deep water and zero current by:

$$\sigma = \sqrt{gk} . \quad (3)$$

Under these conditions, the wave-height energy spectrum $E(\omega)$ as modified by the advective current from its zero-current value $E_o(\omega)$ was found in [6] to be:

$$\frac{E(\omega)}{E_o(\omega)} = \frac{4}{\left[1+\sqrt{1+\frac{4V_C}{g}}\right]^2 \sqrt{1+\frac{4V_C}{g}}} . \quad (4)$$

When the wave and curent velocities oppose each other such that V_C is negative, Eq. (4) can approach infinity as its demonator goes to zero when $\frac{4V_C}{g}$ approaches -1. This physically means that the current velocity and wave group velocity are equal and opposite, implying that standing waves are generated that are very high and steep. This singularity is never reached, because the steepening causes breaking -- a nonlinear dissipation of energy. In the cases examined in this paper (tidal and inertial currents), this singular condition is never reached. However, it explains the increase in wave energy during the part of the wave cycle when currents and waves oppose each other, and decrease when they align. This was seen and discussed with Fig. 2 earlier. See Eqs. (11) and (12) of [7] for derivation of an approximate estimate for the percent fractional height modulation (peak-trough), $2\Delta h$, in terms of the wave height in the absence of modulations, h_o, as:

$$\frac{2\Delta h}{h_o} \approx \frac{\int E(\omega)d\omega - \int E_o(\omega)d\omega}{\int E_o(\omega)d\omega}. \quad (5)$$

A reasonable estimate from Eqs. (5) and (4) is about 0.14 (or 14%). This is in rough agreement with what is seen in our measurements here in Figs. (2) and (4). It is similar to values found in [5] by Masson off Western Canada.

V. CONCLUSIONS

Fixed sensors that monitor ocean waves, such as wave buoys and coastal HF radars, have been seen here and in [5] to output wave parameters -- in particular significant wave height -- containing oscillations produced by modulations from underlying periodic currents. Wave heights observed by SeaSonde HF radars here, and from Masson's buoy measurements in [5] showed periodicity at the M2 semi-diurnal tidal frequency. Our work here at another location off Western Spain shows how oscillations at the 17.9-hour inertial period strongly coupled into both SeaSonde radar and -- to a lesser extent -- into buoy-observed wave height. This is the first, to our knowledge, of radar and buoy observations of these wave modulations, which some initially interpreted as instrument defects or contamination.

These effects are best explained as due to kinematic influences, i.e., changes in the first-order (linear) wave characteristics due to "frequency dispersion." Huang et al. [6] suggested this would suppress the wave heights and steepness when waves and currents propagated in the same direction, and enhance them when opposing. Our Eq. (4) derived in [6] appears to give estimates for percent modulation that agree with observations here and in [5].

Such periodic modulations induced by currents also appear in other wave parameters, including wave period and direction; although examples are not included here, they are shown in our presentation and in [7]. In fact, they are even seen in the radar-observed short Bragg-wave amplitudes used to estimate wind direction [7] and derive currents.

There remain many questions unanswered by these fairly recent discoveries. Although inertial oscillatory modulations were found in both radar and buoy measurements off Spain (Fig. 2), buoy modulations were in phase but weaker than those from the radar. What is the explanation? This is a subject for future investigation.

Among the hypothetical causes might be: (a) The nature of the sensor differences (buoy is a point measurement on the surface, radar is a large-area measurement within a layer near the surface). (b) The wave height spectral model used in radar wave extraction (Pierson/Moskowitz), [7], may not best fit reality in this situation. (c) Underlying currents that are coupling into the second-order spectral echo used by the radar for wave extraction may require a more complex treatment. (d) Likewise, the underlying current acting on the buoy may be biasing its wave-height modulation amplitudes low.

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