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The Gulf of Eilat/Aqaba: a natural driven cavity?

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Motivated by our puzzling high-resolution radar observations of surface vortices in the nearly rectangular Gulf of Eilat/Aqaba, northern Red Sea, we propose and explore the driven cavity approach to this geophysical phenomenon. While the lid-driven cavity has long been considered a benchmark problem in computational fluid dynamics, its oceanographic context has not been considered. Despite the additional effects of rotation and stratification, our modeling demonstrates that when the fluid within a cavity geometrically similar to the Gulf of Eilat is driven by the external current, an interior vortex can develop as in our observations. Furthermore, the Eilat vortices appear only under relatively calm conditions, adding evidence to the intriguing possibility of their simple shear-driven origin.

Keywords: Cavity flow; HF radar; Coherent eddy; Gulf of Eilat

1. Introduction

Beginning in August 2005, our high-resolution ground-based radar observations of flow structures in the Gulf of Eilat/Aqaba, a nearly rectangular basin in the northern tip of the Red Sea (figure 1 and section 2), revealed an occasional and perplexing presence of a large (much of the domain) spatially coherent eddy with a lifetime of a day or so. Such coherent eddies are rare and appear only a few times a year, from November to April, when the wind is relatively calm. The rarity and timing have been puzzling and the mechanism behind the formation of such coherent eddies is unclear. Given the shape of the basin and the strikingly intermittent vortex, we ask whether the vortex could be driven by the external flow at the Gulf opening. In other words, the question is: "Is circulation in The Gulf of Eilat a natural example of a driven cavity?" If so, a geophysical context for the driven cavity problem would provide another link between the fields of physical oceanography and computational fluid mechanics.

Indeed, the driven cavity flow (lid or shear driven) is a class of internal flows, of an incompressible Newtonian fluid which is bounded from all sides and driven by the movement of one of the sides, namely the lid. There are many areas in the industry

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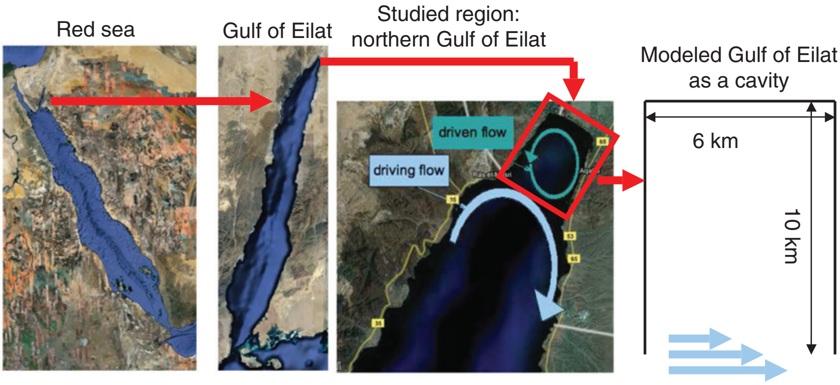

Figure 1. From left to right: The Red Sea, the whole Gulf of Eilat, the studied region (northern end of the Gulf of Eilat), and the model of the studied region as a driven cavity. Note the nearly rectangular shape of the northern part of the gulf. Schematic representation of the "driving current" at the opening and the "driven flow" are indicated by arrows. The red rectangle represents the area modeled as driven cavity. The lid-driven/shear-driven cavity has been a benchmark problem in computational fluid dynamics for decades but here we propose a geophysical context for it (with aspect ratio of 1.6, mimicking the Gulf of Eilat).

where lid driven cavity flows occur (Shankar and Deshpande 2000). The driven cavity has been thoroughly investigated, extensively tested for a wide ranges of Reynolds (*Re*) number and cavity aspect ratio, as well as different cavity boundary conditions including two lid driven and single lid. Lid-driven cavity flows were also tested extensively in the laboratory experiments (e.g. Koseff *et al.* 1983, Koseff and Street 1984, 1985, Prasad and Koseff 1989). The driven cavity is a crucial test case for a new numerical method, since it is a simple geometry, easily meshed, yet combines all the important phenomenon in incompressible fluid mechanics: corner eddies, secondary vortices, change of structure with *Re* number (e.g. Erturk *et al.* 2005b, Shankar and Deshpande 2000). See Shankar and Deshpande (2000) for an extensive review on work done till 2000 (more recent studies include Bouffanais *et al.* 2007, Cheng and Hung 2006, Erturk *et al.* 2005a, Heaton 2008).

Despite the vast literature on driven cavity flow, to the best of our knowledge, no attention has been given to driven cavity flow in geophysical fluid dynamics. In the case of geophysical flow, in addition to the aspect ratio and the *Re* number that govern the classical lid-driven cavity flow, two additional factors should be considered. The first is the effect of Earth's rotation on the flow and the second is stratification. To begin with, both effects can be taken into account in the framework of reduced gravity, shallow water model. Here we present modeling results, showing that under typical condition in the Gulf of Eilat, our observations can be explained via driving by current at the entrance of the Gulf, i.e., shear-driven origin is plausible.

2. The Gulf of Eilat and observed eddies

The northern terminus of the Gulf of Eilat (hereafter "the gulf") is a nearly rectangular, deep ($\sim$700 m), and semi-enclosed basin in the northeast region of the Red Sea (latitude ϕ is 29.5°). The gulf is bounded by desert mountains that steer the typically northerly wind along its main axis (Berman *et al.* 2003). Average wind speed is 4 m s^{-1}

(80% of the time from the north) and net evaporation is approximately 1.6 m per year, ranging from 1 m per year in summer to 3–4 m per year in winter (Ben-Sasson *et al*. 2009).

Cold, dense water from the world ocean cannot flow into the Gulf of Eilat because it is blocked by the shallow sill (137 m) near Bab el Mandeb and the shallow sill (240 m) of the Tiran Strait (Genin 2009). Consequently, stratification across the entire water column in the gulf is relatively weak and deep water forms *in situ*. The weak density stratification breaks down in winter as a result of surface cooling and evaporation, and deep water forms (Wolf-Vecht *et al*. 1992, Genin *et al*. 1995, Biton *et al*. 2008). In February–March we find vertical homogeneity in temperature and salinity reaching a depth of a few hundred meters and sometimes down to the bottom, with new stratification beginning to form in March (Wolf-Vecht *et al*. 1992). In summer the Gulf is stratified with an upper warm layer of up to 200 m depth overlying a homogeneous deeper layer. Accordingly, the first baroclinic Rossby radius changes seasonally, ranging from 6 km to 20 km.

The observed flow field is quite variable and can be rather complex (e.g. figure 2(a)) as the circulation in the gulf is driven by buoyancy, wind, and tides. The gulf is bounded by desert mountains that steer the typically northerly wind along its main axis (Berman *et al*. 2003). The tide is dominated by the semi-diurnal (M2) component, which is peak forced at the Straits of Tiran (Genin and Paldor 1998, Monismith and Genin 2004, Manasrah *et al*. 2006). The tidally-driven flux of water at the strait occupies the layer above the thermocline. From late fall, when the seasonal thermocline deepens, the velocity associated with this flux is significantly reduced (Berman *et al*. 2003, Monismith and Genin 2004).

The configuration and dimensions (nearly rectangular, $10 \times 6\,\mathrm{km}^2$ basin) of the northern gulf enable the observation of surface currents at a high spatial and temporal resolution using high-frequency (HF) radar. A pair of HF 42 MHz SeaSonde stations provide two-dimensional maps of surface currents every 30 min with a spatial resolution of about 300 m. The covered region includes only the northern part of the gulf, down to the widening region of the gulf (figure 1). To reconstruct the velocity at a certain patch of water, at least two radar sites should measure the radial velocity there from two different angles (ideally with at least 15° difference). Because both radar stations are located on the western side of the gulf, they see the southern end from almost the same angle and therefore do not cover the widening region. Since the divergence and vorticity depends on velocity gradients, small error in the measured velocity can lead to large error in the estimates of the vorticity and divergence which should be viewed with caution (Lekien and Gildor 2009). The HF radar systems have been operational in the northern gulf near the city of Eilat, Israel since August 2005. For more details regarding the operation and validation of these measurements, see Gildor *et al*. (2009).

During 4 years of measurements, only on rare occasions, a few times a year from November to April, a coherent eddy occupying most of the region and lasting for a day or so was observed. An example of such eddy is presented in figure 2(b). Although the shape and center of the eddies change in time, they are the dominant feature in the gulf when they are present. The associated velocities are relatively high, and can reach $100\,\mathrm{cm\,s^{-1}}$ near the edge of the eddies, compared to the averaged velocities of $15\,\mathrm{cm\,s^{-1}}$ observed over most of the year. In addition, such eddies appear usually under calm condition, when the wind is relatively weak. The mechanism behind the formation of such coherent eddies is yet unclear and here we explore the idea that it is forced by a

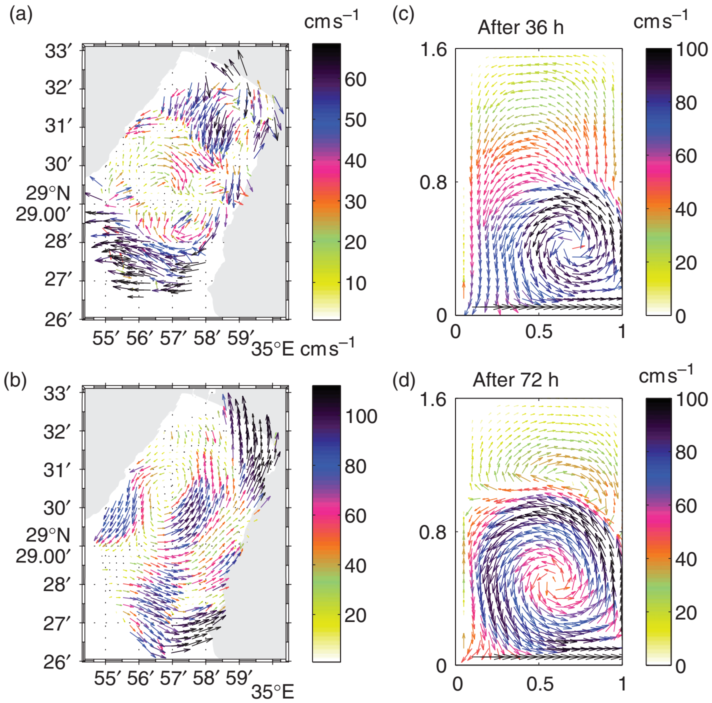

Figure 2. Natural Driven Cavity: Observations and numerical simulations of the coherent eddy. Left panels: Observations of interior vortex: what are the conditions for its occurrence? Snapshots of surface currents measured with the HF radar on two different dates. (a) January 26, 2009, 02:30: no large-scale vortex is present; (b) 3 November, 2009, 05:00: large interior eddy is clearly visible. Large vortex tends to occur under calm conditions, rendering wind-driven origin unlikely. Right panels: Computational evidence for the plausibility of the driven cavity origin for the observed eddies in the gulf. Coherent primary eddy driven by the flow at the opening of the gulf, for $Re = 6000$ and $Fr = 1.15$ (similar eddy appears for a wide range of Re and Fr numbers): (c) after 36 hours; (d) after three days. Hence, the primary eddy geometrically similar to the one observed by our HF radar, can indeed be driven by the flow at the opening of the gulf.

flow past the opening, in a similar manner as primary eddy is formed in a lid-driven cavity flow. Based on numerical simulation using the Princeton Ocean Model forced by climatological wind, Berman *et al.* (2000) suggest that the circulation along the gulf is made up of a series of small scale, localized eddies (sometimes called gyres). Such a gyre near the opening of the northern most 10 km can be the source of the lid-like driving current at the opening of the gulf.

3. Results of computational modeling

The dimensional incompressible reduced-gravity, shallow water equations (when wind and tidal forcing are neglected) of momentum and continuity are

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -g'\nabla h + \nu\nabla^2\mathbf{u} + \mathbf{f} \times \mathbf{u}, \tag{1}$$

$$\frac{\partial h}{\partial t} + \nabla \cdot (h\mathbf{u}) = 0, \tag{2}$$

respectively, where $\nu = \eta/\rho$ is the kinematic viscosity, ρ is the fluid density, η is the fluids' dynamic viscosity, h is the thickness of the upper, active layer, g' is the reduced-gravity, and $\mathbf{f}$ is the Coriolis parameter of magnitude $f = 2\Omega \sin\theta$, where Ω is the rotational velocity of earth and θ is the latitude.

Using the velocity scale U, the depth of the upper layer H, and the length scale L, we define the dimensionless variables $\mathbf{u} = \mathbf{u}'U$, $\mathbf{x} = \mathbf{x}'L$, $t = t'(L/U)$, and $h = h'H$. Nondimensionalizing the above equations, using the definition of reduced-gravity, reveals the non-dimensional parameters – the Reynolds number $Re = UL/\nu$, the Rossby number $Ro = U/Lf$, and the square of Froude number, $Fr^2 = U^2/g'H$. Thus the dimensionless forms of (1) and (2) are

$$\frac{\partial u'}{\partial t'} + (\mathbf{u}' \cdot \nabla)\mathbf{u}' = -\frac{1}{Fr^2}\nabla h' + \frac{1}{Re}\nabla^2 \mathbf{u}' + \frac{1}{Ro}\mathbf{1} \times \mathbf{u}', \qquad (3)$$

where $\mathbf{1}$ is the unit vector in the vertical direction. The Rossby number $Ro = U/Lf$, the Froude number $Fr = U/\sqrt{g'H}$, and the Reynolds number $Re = UL/\nu$ are the parameters of the flow. Usually, ν is taken to be the kinematic viscosity of the fluid. In reality, especially when studying the geophysical processes which span a wide range of spatial scales, the momentum is transferred not by simple diffusion processes (which is slow) but by turbulent motion of small eddies. Therefore, in numerical simulations of the ocean it is common to use "eddy viscosity" to take into account this effect. The specific value of the eddy viscosity is unknown because it depends not only on the fluid but on the flow, and is typically in the range of 0.1–100 $\mathrm{m^2 s}$ in the ocean, depends on the scale of resolved processes.

We solve these equations in a domain with dimensions $10 \times 6\,\mathrm{km}^2$, similar to the northern tip of the gulf but with flat bottom and straight coastlines. We consider no-slip boundary on three sides and constant velocity U of $1\,\mathrm{m\,s^{-1}}$ at the open boundary, analogous to the moving lid. Averaged velocities within the domain are usually 15–20 $\mathrm{cm\,s^{-1}}$ and general circulation model forced by monthly averaged wind yield current at the opening of the gulf of 25–30 $\mathrm{cm\,s^{-1}}$. However, the HF radar observations show that during the eddies events, the velocities are higher than average and can get to 100 $\mathrm{cm\,s^{-1}}$ at certain locations and times within the gyre, hence the driving current should be of the same order.

The spatial discretization is performed on Arakawa C grid (Arakawa and Lamb 1977) with a spatial resolution of 50 m and time-step of 0.2 s. The simulations are started from motionless ocean and the driving current is ramped up in 2 h.

We use the velocity of the lid U, the depth of the thermocline as H, and take the width of the gulf to be length scale L used to define dimensionless independent variables. Based on the density profiles collected at the gulf for the seasons in which the coherent vortices are observed, the typical Fr is $\sim$1 and the Ro number is $\sim$1.

To see whether the observed eddies (e.g. figure 2(b)) can be treated similarly to the driven cavity flow, we run the model for a range of Fr and Re numbers. The circulation pattern after 36 and 72 h for $Fr = 1.15$, and $Re = 6000$ are shown in figure 2(c) and (d). Although the simulation started from motionless ocean, the primary eddy is clearly seen already after 36 hours and after 72 h extend down to more than half the length of the domain. Thus, the time required for the primary eddy development is also consistent with our observations. The spin-down time of the eddy is also O(day) as can be seen in figure 3 which shows the circulation 36 h after the driving current has been turned "off" (note the vector scale).

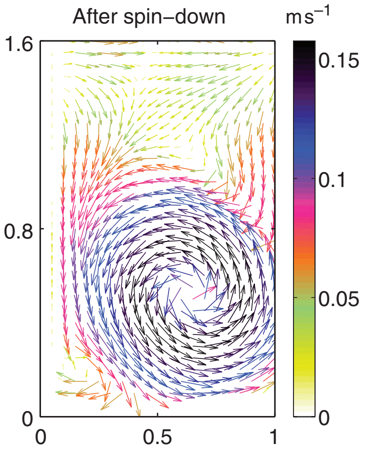

Figure 3. Spin-down of the driven eddy. Computed flow 36 h after the driving current has been stopped completely.

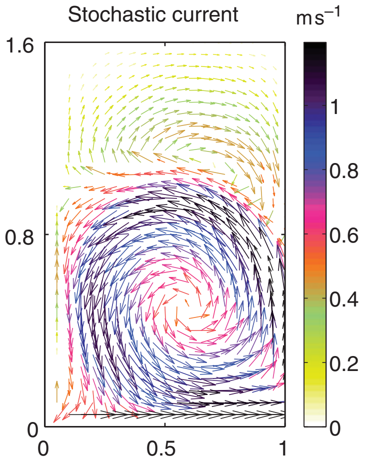

Figure 4. Eddy under stochastic forcing: computed eddy when there driving current is composed off a constant velocity of 1 m s^{-1} plus a stochastic term with a mean of 0.2 m s^{-1}.

In real conditions, the forcing at the open end of the cavity, which in the case of the Gulf of Eilat is the current across the southern boundary, is not steady. We conducted an additional experiment in which we add to the steady current at the entrance of the gulf a stochastic term with an amplitude of 20 cm s^{-1}. The resulting circulation is very similar to the case with constant driving current (figure 4).

We conducted sensitivity tests for a wide range of *Fr* and *Re* and the simulations yield a similar eddy with no qualitative differences, hence the results are not very sensitive to specific choices of the driving current or the stratification. Furthermore, the characteristics of the eddy are not very sensitive to *Ro* number or to the direction of rotation. For very different conditions, e.g. with different aspect ratio, different patterns can evolve, including corner eddies and a secondary vortex, depending mainly on the *Re* number (figure 5).

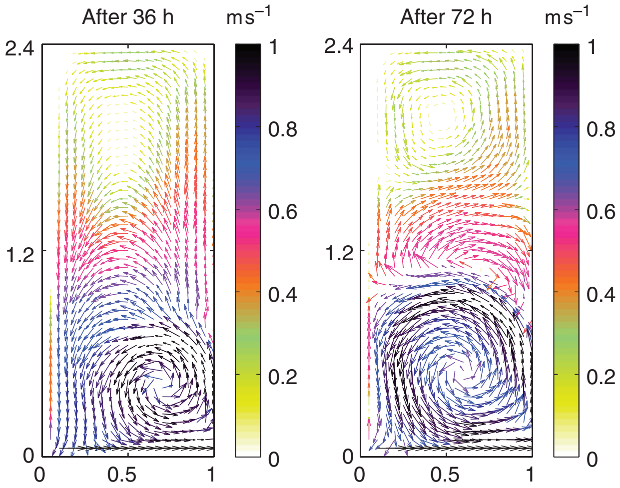

Figure 5. Driven cavity with aspect ratio of 2. (a) After 36 h; (b) after 3 days. All parameters such as *Re* and *Fr* numbers are as in figure 2. With aspect ratio of 2, secondary eddies can be seen.

4. Conclusions

We report on puzzling intermittent observations of a large interior vortex in the gulf and, apparently for the first time, explore the possibility that observed geophysical flow pattern can be treated as driven cavity flow, a benchmark problem in computational fluid dynamics. We use a reduced gravity, shallow-water model within a rectangular domain to explore the possibility that flow pattern observed in the gulf can be treated as driven cavity flow on a geophysical scale. The model shows that a primary eddy with characteristics similar to the one observed by our HF radar, can indeed develop in a reasonable time, assuming that the flow at the entrance drives the flow within the whole domain. In nature, we expect that wind will have a significant influence on the flow. However, because the coherent eddy is observed when winds are weak, our results make it plausible that the observed flow may have a driven cavity origin.

While we used a simplified model and neglected the effects of wind, tides, and irregularities in the coastline or the bathymetry, our study provides the link between the benchmark problem of driven cavity flow and physical oceanography. This link, in turn, opens up the possibility of effects of rotation and stratification on the driven cavity flow. Preliminary results using 3D general circulation model driven by wind stress derived from simulation using a regional atmospheric model for few case studies in which a coherent eddy had been observed, suggest that such eddy can appear even in a more realistic model. These results will be reported in our future studies.

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References

Arakawa, A. and Lamb, V., Computational design of the basic dynamical processes of the UCLA general circulation model. *Methods Comp. Phys.* 1977, **17**, 174–267.

Ben-Sasson, M., Brenner, S. and Paldor, N., Estimating air-sea heat fluxes in semi-enclosed basins: the case of the Gulf of Elat (Aqaba). *J. Phys. Oceangor.* 2009, **39**, 185–202.

Berman, T., Paldor, N. and Brenner, S., Simulation of wind-driven circulation in the Gulf of Elat (Aqaba). *J. Mar. Syst.* 2000, **26**, 349–365.

Berman, T., Paldor, N. and Brenner, S., The seasonality of tidal circulation in the Gulf of Elat. *Isr. J. Earth. Sci.* 2003, **52**, 11–19.

Biton, E., Gildor, H. and Peltier, W.R., The Red Sea during the last glacial maximum: implications for sea level reconstruction. *Paleoceanography* 2008, **23**, PA3219-1–PA3219-8.

Bouffanais, R., Deville, M.O. and Leriche, E., Large-eddy simulation of the flow in a lid-driven cubical cavity. *Phys. Fluids* 2007, **19**, 1363–1376.

Cheng, M. and Hung, K.C., Vortex structure of steady flow in a rectangular cavity. *Comput. Fluids* 2006, **35**, 1046–1062.

Erturk, E., Corke, T. and Gokcol, C., Numerical solutions of 2-D steady incompressible driven cavity flow at high Reynolds numbers. *Int. J. Numer. Methods Fluids* 2005a, **48**, 747–774.

Erturk, E., Corke, T. and Gkl, C., Numerical Solutions of 2-D Steady incompressible driven cavity flow at high reynolds numbers. *Int. J. Numer. Methods Fluids* 2005b, **48**, 747–774.

Genin, A., The physical settings of the Gulf of Aqaba: an explanation for the occurrence of tropical communities in the subtropics. In *Aqaba-Eilat, the Improbable Gulf. Environmental, Biodiversity and Preservation*, edited by F.D. Por, pp. 15–20, 2009 (Isarel: Hebrew University Magnes Press).

Genin, A., Lazar, B. and Brenner, S., Vertical mixing and coral death in the red-sea following the eruption of Mount-Pinatubo. *Nature* 1995, **377**, 507–510.

Genin, A. and Paldor, N., Changes in the circulation and current spectrum near the tip of the narrow, seasonally mixed Gulf of Elat. *Isr. J. Earth. Sci.* 1998, **47**, 87–92.

Gildor, H., Fredj, E., Steinbuck, J. and Monismith, S., Evidence for submesoscale barriers to horizontal mixing in the ocean from current measurements and aerial-photographs. *J. Phys. Oceanogr.* 2009, **39**, 1975–1983.

Heaton, C.J., On the appearance of Moffatt eddies in viscous cavity flow as the aspect ratio varies. *Phys. Fluids* 2008, **20**, 103102.

Koseff, J. and Street, R., The lid-driven cavity flow – a synthesis of qualitative and quantitative observations. *J. Fluids Eng. (transactions of the ASME* 1984), **106**, 390–398.

Koseff, J. and Street, R., Circulation structure in a stratified cavity flow. *J. Hydraul. Eng.–ASCE* 1985, **111**, 334–354.

Koseff, J., Street, R. and Rhee, H., Velocity-measurements in a lid-driven cavity flow. *Mech. Eng.* 1983, **105**, 97.

Lekien, F. and Gildor, H., Computation and approximation of the length scales of harmonic modes with application to the mapping of surface currents in the Gulf of Eilat. *J. Geophys. Res.* 2009, **114**, C06024.

Manasrah, R.S., Al-horani, F.A., Rasheed, M.Y., Al-rousan, S.A. and Khalaf, M.A., Patterns of summer vertical and horizontal currents in coastal waters of the northern Gulf of Aqaba, Red Sea. *Estuarine Coastal Shelf Sci.* 2006, **69**, 567–579.

Monismith, S.G. and Genin, A., Tides and sea level in the Gulf of Aqaba (Eilat). *J. Geophys. Res.* 2004, **109**, C04015.

Prasad, A. and Koseff, J., Reynolds-number and end-wall effects on a lid-driven cavity flow. *Phys. Fluids A* 1989, **1**, 208–218.

Shankar, P. and Deshpande, M., Fluid mechanics in the driven cavity. *Annu. Rev. Fluid Mech.* 2000, **32**, 93–136.

Wolf-Vecht, A., Paldor, N. and Brenner, S., Hydrographic indications of advection/convection effects in the Gulf of Elat. *Deep Sea Res.* 1992, **39**, 1393–1401.