

Fitting Normal Modes to HF Radial and Total Surface Current Vector Data over Enclosed Bays and Estuaries

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ABSTRACT - A technique referred to as Normal Mode Analysis (NMA) has recently been developed for representing total vector CODAR HF radar data in Monterey Bay. These modes satisfy the coastal boundary constraint of no flow normal to the shore, and inherently represent both divergent and rotational flow as two sets of ortho-normal basis functions. In prior investigations by others, the domain had a large open boundary at which additional information from a numerical model was needed to completely represent the surface flows in the Bay. The modes were fitted to data only in the two-site overlap region where total vectors were calculated. We apply NMA for completely enclosed bays, using two-dimensional finite element methods to derive these modes where the shoreline is the mathematical boundary for the problem. This is an improvement over prior studies with open boundaries where additional information was needed to represent the flow within.

We also extended this methodology by fitting to radial velocities from each radar by itself -- as well as simultaneous radial data from multiple coastal radars viewing the same bay. It was applied to Corpus Christi Bay where two SeaSondes have been operated by Texas A&M University for two years. First, we employed simulations, where we resolved arbitrary current flow patterns into two sets of radial data. Noise was added to the vectors, and the extraction accuracy was studied. Ability to derive meaningful total velocity patterns depends on the noise level; the percent coverage of the bay by the radial measurements; and the availability of simultaneous radial data from different sites/angles. Surprisingly good extraction is often obtained with only single radar coverage. Finally, this method is now tested here with actual, hourly SeaSonde HF radar data over this Bay, both at the single-site radial level and by employing both sites. Comparisons are made with the real-time total-vector maps produced by the radar software over the common coverage area. Our bay-conforming natural mode-pattern resonances will be used in ongoing studies that relate their strengths to wind stress across the bay surface.

I. INTRODUCTION

The analysis of the surface currents and related kinematic and dynamic properties for Corpus Christi Bay has been done using High Frequency (HF) radars to measure surface currents from the Doppler shifts of radar sea-scattered echoes. Once extracted, this data from two or more radars is used to create hourly maps of the total surface vectors that accurately map the surface currents of the bay. However, the mapping of the surface currents using this method is limited by the number of radars used and the topography of the bay or estuary in question, as only the overlapping fields of unobstructed coverage can be used to plot two-dimensional surface vectors from the radars being used. In the case of Corpus Christi two CODAR HF radars are implemented to map out the majority of the bay. But how can one extend mapping to cover the entire bay with the data sets given? Building on work done by Lipphardt et al. [1] for Monterey Bay, CA we use Normal Mode Analysis (NMA) to produce a reasonable maps for Corpus Christi Bay using one or more HF radar data sets without having to constrain only to the area over lapped by both radars.

II. NORMAL MODE ANALYSIS

We use a partial differential equations model (PDE2D) created by Sewell [2] to produce two sets of hydrodynamic NMA basis functions called for in [1]. We use these basis functions to fit the one-dimensional scalar radial components directly from the CODAR HF radar hourly data to estimate a two-dimensional flow pattern inherent to the coastline-conforming NMA mode functions. As presented by Eremeev et al. [3] and Lipphardt et al. [1], an incompressible 3-D velocity field in water can be constructed by the use of two scalar potentials then solved for two cases: homogeneous and inhomogeneous. The homogeneous case constrains the velocity field to the closed coastline of the bay, which constitutes Dirichlet and Neumann boundary conditions within the PDE2D model. The inhomogeneous case solves for a velocity field due to the specified normal flow through the bay's open boundaries.

The PDE2D model thus represents the interior homogeneous velocity field as an eigenfunction expression of two sets of NMA basis functions that describe the horizontal divergence and relative vorticity for process studies. The horizontal divergence is represented by a set of basis functions that satisfy the Neumann boundary conditions and the vorticity is represented by a set of Dirichlet modes. We will call these two solution sets velocity potential and stream function modes, respectively, from now on. In the studies previously undertaken by Fitzgerald et al. [4], PDE2D produced approximately eight significant stream functions and eight velocity potential modes for a total of 16. With these we now proceed to develop a method in which we approximate the two-dimensional vector maps for the Corpus Christi Bay.

III. METHODOLOGY

To make direct fits to HF radar data we first calculate radar-directed scalar components of the total mode velocities for each basis function. PDE2D produced u and v components of the calculated velocity for each mode on a one-by-one kilometer grid covering the bay, with an origin to be user selected. We then find the radar-directed components of the velocity at our Cartesian grid points by finding the inner product of the u and v velocities based on measured radial velocities on the radar polar grid:

$$\vec{V}_{i,j}^{r,n} \equiv \hat{r}_{i,j} \left[u_{i,j}^n \hat{x} + v_{i,j}^n \hat{y} \right]$$

Here r denotes the radial component; n is the mode being considered; j and i are indicate which Cartesian coordinate point is being used. We now can express the model as a series of radar-directed Normal Mode velocity functions times unknown coefficients.

Now that the radar-directed velocities are known for each mode we then interpolate the nearest raw radial velocity data for each radar onto the PDE2D model coordinate system. In order to proceed we find the average of the nearest radar data points to each model data point up to a maximum of four bracketing radar data points per model point. Once we have this data set of averaged values we use the method of linear least squares to fit the averaged values to the model data.

IV. LEAST SQUARES APPLIED TO OUR PROBLEM

Given an overdetermined system of equations, use of the least squares minimization leads into a completely determined system with an invertible square matrix. The system of equations comprises the summation of our 16 normal mode functions aforementioned. For Corpus Christi Bay PDE2D uses up to 728 data values (at each grid point

where there is data) for each of the 16 modes (whose coefficients are unknown), thus giving us a very overdetermined system. We can define the system of equations as the matrix equation:

$$[\mathbf{b}_n] = [\mathbf{a}_{n,k}] [\mathbf{x}_k]$$

Here \mathbf{b} is the averaged radar data at the model point; n , k indicates the normal mode in question at that point. The matrix, \mathbf{a} represents the radar-directed components of the normal modes for each point corresponding to \mathbf{b} and \mathbf{x} represents the mode coefficient to be determined by least-squares fitting. There are N equations expressed as:

$$B_k \equiv \sum_{n=1}^N A_{k,m} x_m \quad 1 \leq n \leq N$$

To minimize the mean-squared differences between the data points, \mathbf{b} and the fitting function $[\mathbf{a}][\mathbf{x}]$ the following expression was employed.

$$L(\bar{x}) = \sum_{n=1}^N \left[b_n - \sum_{k=1}^M a_{n,k} x_k \right]^2 \Rightarrow \text{Minimum}$$

Here L is referred as the residual fit, which would be zero if the fit were perfect, but can never be zero for more equations than unknowns with noisy data. To find the minimum of the residual fit it must be differentiated with respect to its variables and set to zero. The m -th equation can be expressed as:

$$\sum_{n=1}^N b_n a_{n,m} = \sum_{n=1}^M \sum_{k=1}^N a_{n,k} a_{n,m} x_m$$

Within this expression there are now M equations with M unknowns, a square system of linear equations. The following expressions can now be defined as:

$$A_{k,m} \equiv \sum_{n=1}^N a_{n,k} a_{n,m}; \quad B_m \equiv \sum_{n=1}^N b_n a_{n,m}$$

The new system of M equations and M unknowns can now be expressed as:

$$B_k \equiv \sum_{n=1}^N A_{k,m} x_m$$

V. RESULTS

We now apply these equations to determine the fitted coefficients for the bay-conforming modes given earlier by the PDE2D model. We then compare total velocity map outputs to the total vector maps originally produced by the CODAR software for the common area where the two radar's coverages overlap. Below is a comparison of typical NMA-fitted hourly data (Fig. 1) with corresponding total vector data from CODAR SeaDisplay® software (Fig. 2). It is important to note that the fitted map included radial data from *only one* radar, the one shown as the red dot on the South coast of the Bay of Fig. 2. In other words, the simultaneous radial data from the site to the West were withheld. As can be seen, the agreement in the measurement area of Fig. 2 is better than expected.

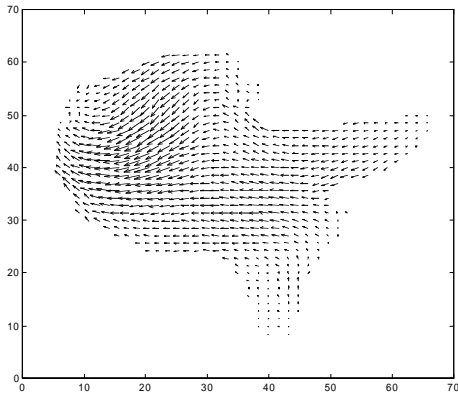


Figure 1. NMA-fitted total vector map of Corpus Christi Bay, 0700 August 29, 2001, using single radar

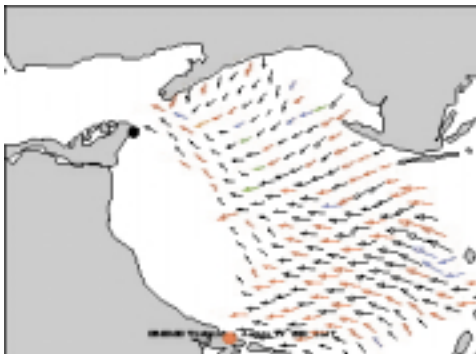


Figure 2. Total-vector "sea-truth" map from CODAR SeaDisplay software for Aug, 29 2001 0600-0700 hrs

Next, we compare least-squares-fitted NMA estimates using *both* radars with the above SeaDisplay map.

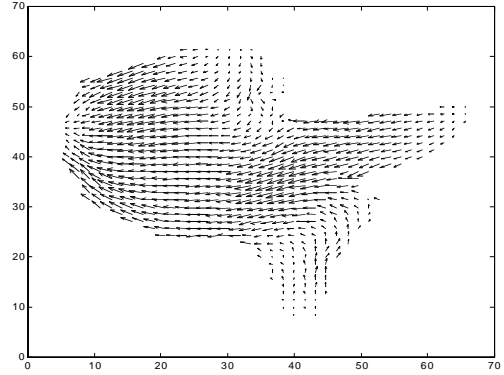


Figure 3. Fitted total vector map of Corpus Christi Bay 0700 August 29, 2001 using data from both radars

Table 1 shows the RMS error measures for the differences between input radar radial velocities and the NMA fitted radials at the same points for each radar separately. Radar 1 is to the West and Radar 2 is on the South shore of the bay.

	Radar1	Radar2
RMS of radial velocities	5.91167	5.16122
RMS of radial differences or errors	3.52815	1.94569
Ratio of above RMS values	0.596812	0.37698

VI. CONCLUSIONS

We have used Normal Mode Analysis to fit total velocity map estimates to radial velocities over Corpus Christi Bay measured by a pair of SeaSonde HF radars. We have also shown that it is possible to generate accurate vector maps over the bay utilizing only one radar data set instead of both. Calculating the total velocity maps in the common overlap area between two radars had represented the "conventional wisdom" up to now. We have just begun detailed quantitative comparisons of these fitted-mode data maps with "truth", i.e., the actual total velocities in the overlap region. This new method of characterizing and extending surface current maps in bays and estuaries should aid many applications in environmental studies as well as possible defense and civilian rescue capabilities.

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VIII. REFERENCES

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