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Inversion of Second-Order Radar Echoes From the Sea

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The Doppler spectrum of high-frequency radar echoes from the sea consists of dominant peaks due to first-order Bragg scatter surrounded by a higher-order continuum. Most applications to date have been based on the first-order lines, requiring multiple observations and large or moving antennas. In contrast, inversion of the second-order structure can yield the complete directional ocean wave spectrum from a single radar observation. In this report we describe the first inversion of measured second-order echo spectra from a 21.75-MHz narrow-beam radar looking in a single direction. Estimates of the directional ocean wave spectrum are compared with surface truth provided by tilt buoy and weather station, and fair agreement is found. This initial success is indicative of the potential of this technique for remote sensing of the sea surface.

INTRODUCTION

The Doppler spectrum of a high-frequency radar signal backscattered from the sea surface has been used to give detailed information on the properties of ocean waves (see, for example, *Teague et al.* [1977]). Most applications depend on interpretation of the dominant first-order lines arising from simple Bragg scatter. In this mechanism the radar wave interacts only with ocean waves of one half the radar wavelength moving directly toward or away from the receiver; hence for detailed information on amplitude and directional characteristics one requires observations at several look angles and transmitter frequencies. This is often impractical, owing to limitations on radar design and difficult propagation conditions. A further difficulty is that for the observation of the important lower end of the gravity wave spectrum using the first-order lines the radar frequency must be in the band 50 KHz to 2 MHz, requiring large arrays or moving antennas for the formation of a directional beam. However, the first-order lines are surrounded by a continuum due to higher-order Bragg scatter which involves ocean waves of all angles and wavelengths and therefore can in principle yield the directional ocean wave spectrum from a single Doppler spectrum. Observations of second-order structure show that it is a sensitive function of sea state [*Tyler et al.*, 1972]. *Hasselmann* [1971] first suggested that the second-order Doppler side bands should be proportional to the wave height nondirectional spectrum. He also suggested a convenient normalization in which the side bands are divided by the first-order echo, canceling unknown factors such as path loss and system gains. The relationship between the second-order structure and the directional ocean wave spectrum was derived by *Barrick* [1972] and has the form of a two-dimensional nonlinear integral equation. *Barrick* [1977*a*] derived an approximate closed form technique to obtain the wave height nondirectional spectrum from the second-order structure. Wave height and wave period predicted by using this solution have been found to agree with simultaneous buoy observations [*Barrick*, 1977*b*]. *Lipa* [1977] has derived and discussed a method of integral inversion of the second-order spectrum. This paper describes the first application of this technique to measured data. Derived parameters define the directional distribution of saturated ocean waves and the nondirectional spectrum of unsaturated waves and are found to be in good agreement with surface truth observations provided by a tilt buoy. Inversion of the second-order structure using this method requires radar frequencies in the upper HF band, suitable for sky wave propagation to great distances. The method can therefore be applied to sky wave spectra, if the effects of ionospheric distortion can be eliminated.

INVERSION METHOD

The second-order structure arises from the interaction of the incident ocean wave with two ocean waves to yield four scattered waves which satisfy the second-order Bragg condition. For backscatter of vertically polarized radar of wave number β the radar cross section at a Doppler shift η is given by

$$\sigma(\eta) = 2^6\pi\beta^4 \sum_{m_1, m_2 = \pm 1} \iint_{-\infty}^{\infty} |\Gamma|^2 \delta[\eta - m_1(g\kappa_1)^{1/2} - m_2(g\kappa_2)^{1/2}] S(m_1\tilde{k}_1) S(m_2\tilde{k}_2)\, dp\, dq \qquad (1)$$

where the scattering ocean wave vectors are given by $\tilde{k}_1 = (p - \beta), q$, $\tilde{k}_2 = -(p + \beta), -q$, the ocean wave numbers are $\kappa_1 = |\tilde{k}_1|$, $\kappa_2 = |\tilde{k}_2|$, and the Bragg condition is imposed by the delta function constraint, where g is the gravitational acceleration, Γ is the transfer coefficient, which is the coherent sum of an electromagnetic and a hydrodynamic term [*Barrick*, 1972], and $S(\tilde{k})$ is the nonsymmetric directional ocean wave spectrum. We seek to determine sufficient parameters to define $S(k)$ by inverting (1). As the number of parameters sought from a given data set is increased, the accuracy is correspondingly decreased. In this first attempt at inversion of a second-order radar spectrum we have assumed that the spectrum is the product of independent factors, an amplitude spectrum dependent only on wave number and a directional factor which is a function of angle relative to the wind:

$$S(\tilde{k}) = F(\kappa)G(\theta) \qquad (2)$$

With this assumption of separability, fewer parameters are required to define the ocean wave spectrum; the parameter set is further reduced by assuming symmetry about an undefined direction, to be determined by the inversion procedure. These approximations apply to short saturated waves when the directional distribution is approximately independent of wavelength, but faster waves may have a narrower spread about the wind direction (see, for example, *Tyler et al.* [1974]). These long waves affect the radar frequency spectrum in the region surrounding the first-order Bragg line, which for high winds and/or radar frequencies is relatively insensitive to the directional distribution of ocean waves as discussed by *Barrick* [1977*a, b*]. Results of inversion will then be insensitive to the breakdown of assumption (2) for long waves.

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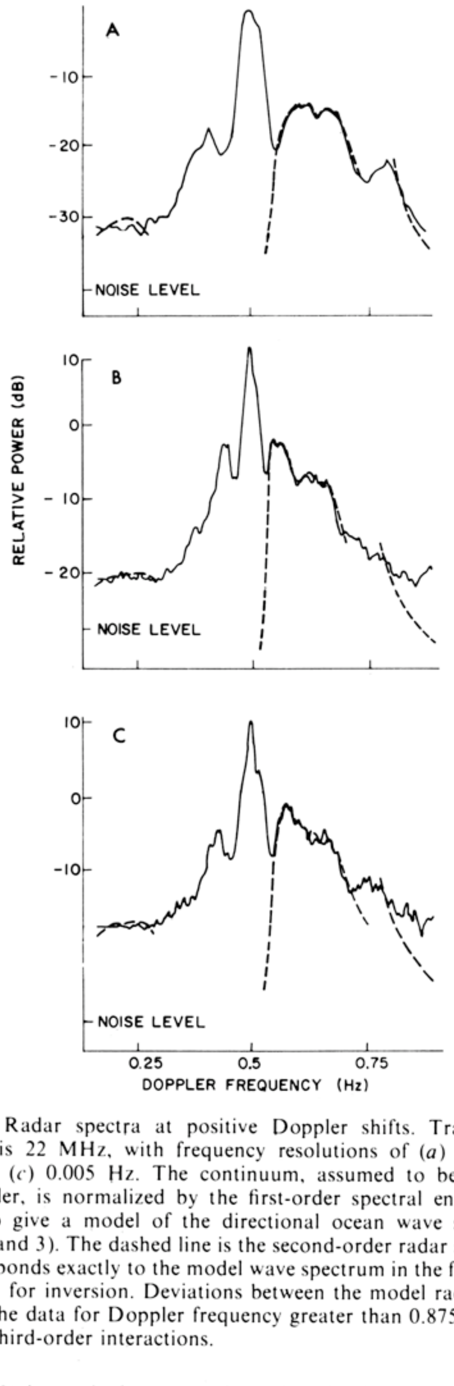

Fig. 1. Radar spectra at positive Doppler shifts. Transmitter frequency is 22 MHz, with frequency resolutions of (*a*) 0.01, (*b*) 0.005, and (*c*) 0.005 Hz. The continuum, assumed to be entirely second order, is normalized by the first-order spectral energy and inverted to give a model of the directional ocean wave spectrum (Figures 2 and 3). The dashed line is the second-order radar spectrum that corresponds exactly to the model wave spectrum in the frequency bands used for inversion. Deviations between the model radar spectrum and the data for Doppler frequency greater than 0.875 Hz may be due to third-order interactions.

When (2) is used, the spectral product in the integral equation becomes a quartic; for example, for m_1, m_2 equal to + 1,

$$S(\tilde{k}_1)S(\tilde{k}_2) \qquad F(\kappa_1)G(\theta_1)F(\kappa_2)G(\theta_2) \tag{3}$$

The solution of the inversion problem is simplified by the fact that the ocean wave number κ_2 is always greater than the radar wave number and for typical radar frequencies corresponds to a saturated wave. We may therefore substitute for $F(\kappa_2)$ the saturated amplitude spectrum [*Phillips,* 1966].

$$F_{\text{sat}}(\kappa) \propto 1/\kappa^4 \tag{4}$$

We then estimate the amplitude spectrum and the directional factor separately using different regions of the spectrum. In the frequency bands separated from the first-order Bragg frequency, both of the contributing ocean waves are saturated with an amplitude spectrum given by (4). The spectral factor

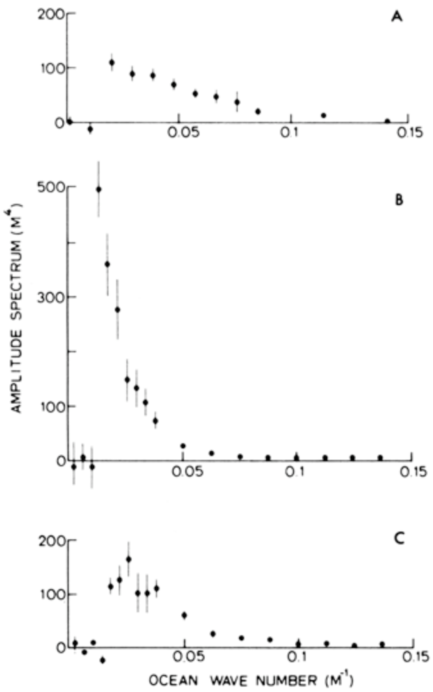

Fig. 2. Amplitude spectra obtained by inversion of radar spectra shown in Figure 1. Error bars represent one standard deviation and are computed from the variance in wave height of the random sea surface.

(3) then reduces to a quadratic function of the directional factor, and the integral equation may be reduced to a set of quadratic equations which are solved by iteration for parameters of $G(\theta)$, including the direction of symmetry.

To derive the amplitude spectrum of the unsaturated waves, we use the spectral points with frequencies slightly greater than the first-order Bragg frequency. In this region the shape of the spectrum is almost independent of ocean wave directional properties. Substituting the derived function $G(\theta)$ and the saturated amplitude spectrum for $F(\kappa_2)$ linearizes the spectral factor (3). It is then straightforward to reduce the integral equation to a set of linear equations which may be solved for parameters of the unsaturated amplitude spectrum.

This inversion method is described in detail by *Lipa* [1977].

DATA AND INVERSION RESULTS

The radar data, provided by the Stanford Center for Radar Astronomy, were measured on the California coast by using a coherent pulse Doppler radar [*Teague et al.,* 1977]. The transmitting antenna was a vertical half rhombic approximately 250 m long and 45 m high at the apex, supported by a helium-filled balloon, and the backscattered signal was received by a wideband loop antenna. The radar beam pointed northwest into the direction of the prevailing winds with a half power beam width of 10° at 22 MHz. Adequate spectra for testing the inversion method were available only in downwind conditions,

TABLE 1. Characteristics of Temporal Spectra

Spectrum	Peak Period, s	rms Wave Amplitude, m
A	8 ± 0.5	0.61 ± 0.09
B	14 ± 1, 12 ± 0.5	0.43 ± 0.04
C	13 ± 1, 11 ± 0.5	0.46 ± 0.04

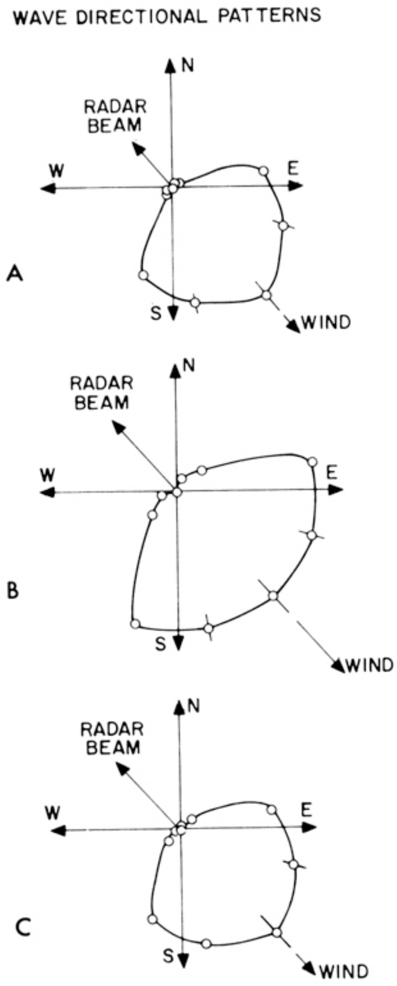

Fig. 3. Directional factors obtained by the inversion of radar spectra shown in Figure 1 for ocean waves of periods of <4.5 s. The model ocean wave spectrum we adopt is the product of the directional factor and the amplitude spectrum.

as high winds in the location normally blow from the northwest, and in addition, the antenna is difficult to control in crosswind conditions. Transmit antenna motion causes a broad spectral hump in the region around zero Doppler shift, which then cannot be used to derive directional information.

TABLE 2. Comparison of Radar and Buoy Observations

	Radar	Buoy
rms wave amplitude, m	0.61 ± 0.1	0.54
Wind direction, deg	312 ± 5	310
N_{10}/N_{00}	−0.56 ± 0.1	−0.59*
N_{01}/N_{00}	0.51 ± 0.09	0.49*
N_{20}/N_{00}	0.51 ± 0.09	0.61*
N_{02}/N_{00}	0.49 ± 0.08	0.58*
N_{11}/N_{00}	−0.12 ± 0.03	−0.19*

*Average for ocean wave period between 1.9 and 4.5 s.

Radar spectra for positive Doppler shifts are shown in Figure 1. Spectra A, B, and C were measured on July 25, October 21, and October 23, 1975, with frequency resolutions of 0.01, 0.005, and 0.005 Hz, respectively. Incoherent averaging was performed over independent spectra measured over a period of 2 hours and four range bins 4 km in extent. Spectra which indicated severe antenna motion were not included in the averaging. Before inversion the spectra were normalized by the first-order energy, obtained by integrating over the first-order region of the spectrum. First-order terms were then subtracted to give the higher-order continuum with a cutoff near the first-order Bragg frequency. The major source of variance in the data is the random surface height of the sea. For each parameter derived by inversion we calculated a corresponding standard error resulting from this source of uncertainty, which decreases with the number N of spectra averaged as $1/(N)^{1/2}$.

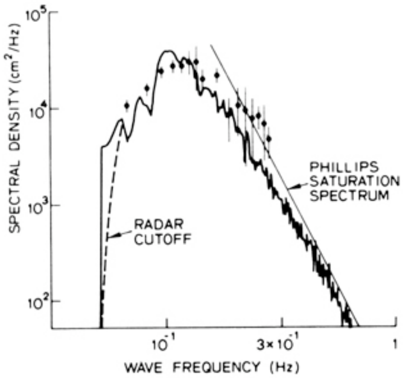

Fig. 4. Comparison of the temporal wave height spectrum obtained by inversion of the radar spectrum shown in Figure 1*a* with that measured by the buoy (continuous line).

The amplitude spectra derived by inverting the radar spectra for frequencies between 0.5 and 0.75 Hz are shown in Figure 2. Spectrum A is characteristic of a moderate well-developed sea, while spectra B and C contain peaks due to long-wavelength low-amplitude swell. The derived peak periods and rms wave amplitudes are given in Table 1.

Derived directional distributions for waves with periods less than 4.5 s are shown in Figure 3. In each case the symmetry direction is calculated to coincide with the radar beam, and most energy propagates within 60° of the wind. It is of interest to note that the popular $\cos^s(\theta/2)$ model for the directional factor provides a poor fit to the derived functions for any value of the spread parameter s. A far better fit is provided by a model defined by

$$G(\theta) = \text{const} \qquad \theta < \theta_c$$

$$G(\theta) = 0 \qquad \theta > \theta_c$$

where θ_c defines a cutoff angle, in our case, 60°.

ESTIMATION OF SUCCESS OF INVERSION

The derived parameters constitute a model of the directional ocean wave spectrum. We consider the inversion of the linearized integral equation to be successful if the following criteria are obeyed:

1. The derived parameters uniquely satisfy the linear equations defined by (1); we find that this criterion is obeyed as the system is overdetermined.
2. The uncertainties in the model due to the variance of the data are reasonably low: the error bars shown in Figures 2 and 3 are considered acceptable for many applications.
3. The model agrees with available surface truth.
4. The model is compatible with the data.

We shall consider the third and fourth points separately.

Comparison with surface truth. For spectrum A in Figure 1, simultaneous tilt buoy observations provided by Scripps Institute of Oceanography are available. We note that the radar provides an areal average over approximately 50 km², while the buoy gives a point measurement, both devices averaging over time. The tilt buoy measures the first six Fourier coefficients of the ocean wave spectrum $S(\kappa, \theta)$, given by

$$N_{pq}(\kappa) = \int_0^{2\pi} \cos^p(\theta) \sin^q(\theta) S(\kappa, \theta)\, d\theta \qquad (5)$$

$$p, q = 0, 1, 2 \qquad |p + q| \leq 2$$

[*Tyler et al.*, 1974]. Of these, five are independent as $N_{00}(\kappa) = N_{02}(\kappa) + N_{20}(\kappa)$. $N_{00}(\kappa)$ is just the spatial spectrum of the ocean waves, which may be compared with that derived by inversion of the radar spectrum. Results are shown in Figure 4, where the spatial spectra have been converted to temporal spectra by using the first-order dispersion relation for gravity waves. We consider agreement between the two observations to be fair, considering that they were obtained by devices which perform different types of averaging.

The next five Fourier coefficients measured by the buoy were normalized by $N_{00}(\kappa)$. The normalized values are approximately constant for wave periods of less than 4.5 s and were averaged to give values for comparison with the radar results, derived by integrating the directional factor shown in Figure 3*a*. Results are given in Table 2, which also contains corresponding rms wave amplitudes and mean wave directions. Errors quoted in Table 2 represent upper bounds on the uncertainties.

For spectra B and C no buoy observations were available. According to the Farallons weather station the wind at the time was from the northwest at 18 and 12 kn (9 and 6 m/s), respectively, with swell of height 5 ft (~1.5 m) and a sea height of 2 ft (~0.6 m) for both B and C. These observations are consistent with our results.

Compatibility with the data. The derived model of the ocean wave spectrum was substituted into the right-hand side of (1), and the corresponding radar spectrum was calculated. A measure of the compatibility of the model wave spectrum with the data is then given by the deviation of the computed radar spectrum from the measured values. For spectrum A the fit is good throughout the whole frequency band. For B and C it is good except for frequencies greater than 0.875 Hz, where model values are considerably less than the data. We draw the conclusion that third-order terms in the perturbation expansion may become significant in this region. This conclusion is supported by model calculations indicating third-order peaks for large Doppler shifts [*Lipa and Barrick*, 1977].

CONCLUSION

We have demonstrated that inversion of the second-order radar spectrum yields detailed information on the directional ocean wave spectrum. The inversion method described derives the amplitude spectrum of long ocean waves (wavelength of >3 m) and the directional factor of shorter saturated waves, together with the direction of maximum symmetry which may be identified with the wind direction. Ocean parameters derived from a measured echo spectrum are found to be in good agreement with concurrent observations made by a tilt buoy. This success lends impetus to the development of methods to derive the complete directional spectrum by inversion of the second-order structure and the assessment of limits of application set by contamination with third-order and viscous terms.

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