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Theory of HF and VHF propagation across the rough sea, 1, The effective surface impedance for a slightly rough highly conducting medium at grazing incidence

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One method of analyzing radiation and propagation above a surface is to employ an effective surface impedance to describe the effect of the boundary. In this paper an expression is derived for the effective impedance at grazing incidence of a slightly rough interface between air and a finitely conducting medium for which the Leontovich boundary condition is applicable. The perturbation technique of Rayleigh and Rice is employed, and attention is restricted to vertical polarization. The resulting effective surface impedance consists of two terms, the impedance of the lower medium when the surface is perfectly smooth and a term accounting for roughness. The latter term can be complex in general and depends on the strengths of the roughness spectral components present. The result is applicable to either deterministic periodic surfaces or random rough surfaces. Various alternate definitions of the effective surface impedance are examined and are seen to be equivalent. The analysis of power flow at the surface permits the interpretation of the interaction process in terms of scattered fields.

INTRODUCTION

The development of radio at the turn of the century heralded an interest in understanding the mechanism of propagation over the earth; the classic works of *Sommerfeld* [1909] and *Norton* [1936, 1937*a, b*] clarified the nature of radiation from a dipole above a planar, highly conducting earth. *Van der Pol and Bremmer* [1937, 1938, 1939] and *Fock* [1945] obtained asymptotic solutions to the problem of radiation above a smooth spherical earth, and *Norton* [1941] generated curves to facilitate the use of these results. No less than five hundred papers have appeared on this subject over the past 70 years; for a more nearly complete review of the history, the reader is referred to a thorough summary by *Wait* [1964] and to a discussion and bibliography by *Barrick* [1970].

Nearly all treatments of the subject took the earth surface as a perfectly smooth (planar or spherical) interface between the air and ground or water. *Feinberg* [1944] published a result derived from an integral equation formulation of the problem of radiation above a planar earth with terrain irregularities. He showed that the effect of small height irregularities was to decrease the apparent conductivity of the earth. Details of his derivation were not provided, and his result did not show the effect of finite earth conductivity; the term accounting for roughness was obtained as though the surface were perfectly conducting. *Bremmer* [1958] repeated Feinberg's result. *Rice* [1951], in a classic treatment of scattering from slightly rough random surfaces, analyzes the problem of propagation along an irregular perfectly conducting surface by employing a Fourier series expansion for the surface and a plane-wave summation for the scattered waves. *Wait* [1959] derives an effective impedance for a perfectly conducting surface upon which are distributed hemispheric bosses of constant radius; the approach taken is attributed to *Twersky* [1951]. *Senior* [1960] derived a general expression for the influence of a slight roughness on the surface impedance, but his results are not simplified sufficiently to permit analysis of the pathological region near grazing incidence.

It has long been known that a slight rectangular corrugation on an otherwise perfectly conducting surface has a marked influence in the guiding or trapping of waves near the surface; surface-wave antennas are built on this principle. When both the height and spatial period of the corrugations are small in terms of a wavelength, expressions have been derived for the inductive contribution to the surface impedance [*Wait*, 1957; *Barlow and Brown*, 1962]. The sea surface at HF and VHF can have waves whose heights are an appreciable part of a radio wavelength; hence, one should expect to see trapping effects similar to those obtained for corrugated surfaces.

In this paper we derive a result for the effective impedance at grazing incidence of a slightly rough surface electrically describable by the Leontovich (or impedance) boundary condition. The basic technique used is that of *Rice* [1951]. When we allow the impedance of the material below the surface to vanish (i.e., it becomes perfectly conducting), our expression checks with those of Feinberg and Rice. Rice did not actually define an effective surface impedance in his analysis. We show how it is possible to define this effective impedance in either of two physically meaningful ways; both definitions give equivalent results. By examining the spatially-averaged Poynting vectors near the surface, we will interpret this effective impedance in terms of scattered and evanescent modes. Results are derived for deterministic periodic surfaces as well as random rough surfaces.

In this paper we describe the fields near the surface in terms of guided waves. In paper 2 we show how the effective impedance derived using a guided wave approach can be utilized to analyze the problem of radiation from a dipole located above such a surface. As an application, we numerically compute the effective surface impedance for the sea by using two different directional, empirical models for the ocean wave-height spectrum. Finally, we employ these surface impedances in calculations to predict the basic transmission loss for propagation across the sea as a function of sea state, frequency, and antenna heights.

The significance and convenience of the normalized surface impedance in the formulation of ground-wave propagation problems were emphasized and popularized by *Wait* [1964]. This dimensionless quantity is defined as $\Delta \equiv Z_s/Z_0$, where Z_s is the impedance of the surface in ohms and $Z_0 \simeq 120\pi\ \Omega$ is the impedance of free space. For the earth and sea at frequencies up to VHF, Δ is much less than unity, and hence vertical polarization is favored for ground-wave propagation. The normalized impedance at grazing for a smooth interface with a lower medium of complex permittivity ϵ_1 and permeability μ_1 can be written [*Wait*, 1964] as

$$\Delta = \frac{1}{120\pi}\left(\frac{\mu_1}{\epsilon_1}\right)^{1/2} \times \left(1 - \frac{\epsilon_0\mu_0}{\epsilon_1\mu_1}\right)^{1/2} \simeq \frac{1}{120\pi}\left(\frac{\mu_1}{\epsilon_1}\right)^{1/2}$$

for $\epsilon_1 \gg \epsilon_0$. But $\epsilon_1 = \epsilon_{r1} + i(\sigma_1/\omega_0)$ (the time convention $e^{-i\omega_0 t}$ is employed), where ϵ_{r1} is the real permittivity and σ_1 is the conductivity of the medium. If we take as an example sea water at 10 MHz, we have $\epsilon_{r1} = 80\ \epsilon_0$, $\sigma_1 \simeq 4$ mho/m, and hence $\Delta \simeq 1.18 \times 10^{-2} \exp(-i\pi/4)$. Thus $|\Delta|$ is truly small for sea water at this frequency; nevertheless, it will be shown that it is not negligible when the effect of roughness is analyzed.

Inherent in the analysis to follow are several assumptions and approximations. These we state here as: first, the surface height ζ above a mean plane is small in terms of the wavelength λ; i.e., $(k_0\zeta)^2 \ll 1$, where $k_0 = 2\pi/\lambda$ is the free space wave number; second, the surface slopes ζ_x and ζ_y ($\zeta_x = \partial\zeta/\partial x$, $\zeta_y = \partial\zeta/\partial y$) are small for the roughness waves considered here; and third, the medium below the surface is highly conducting; i.e., $|\Delta| \ll 1$. The first assumption guarantees the applicability of the Rayleigh technique, but recent studies [*Burrows*, 1969] show that the Rayleigh hypothesis can be applied even when assumption 1 is not satisfied; we shall nonetheless specify the regions of validity of our results in terms of these three restrictions. The satisfaction of all three restrictions also ensures the applicability of the Leontovich boundary condition.

DERIVATION OF EFFECTIVE SURFACE IMPEDANCE

Fourier expansion for surface height. The *Rice* [1951] technique to be followed here (the notation of Rice will be retained everywhere to facilitate reference to his paper) begins with the representation of the surface height $\zeta(x, y)$ above the $z = 0$ plane in terms of a Fourier series periodic over a square of side L: (see Figure 1)

$$\zeta(x, y) = \sum_{m,n=-\infty}^{\infty} P(m, n) \exp\left[ia(mx + ny)\right] \qquad (1)$$

where $a = 2\pi/L$ and $P(m, n)$ are the Fourier coefficients of the surface expansion. We assume that the mean plane is the $z = 0$ plane so that $P(0, 0) = 0$. Furthermore, since $\zeta(x, y)$ is a real number, $P^*(m, n) = P(-m, -n)$, where P^* denotes the complex conjugate.

When the surface height is a nonperiodic random variable, the above series representation can still be

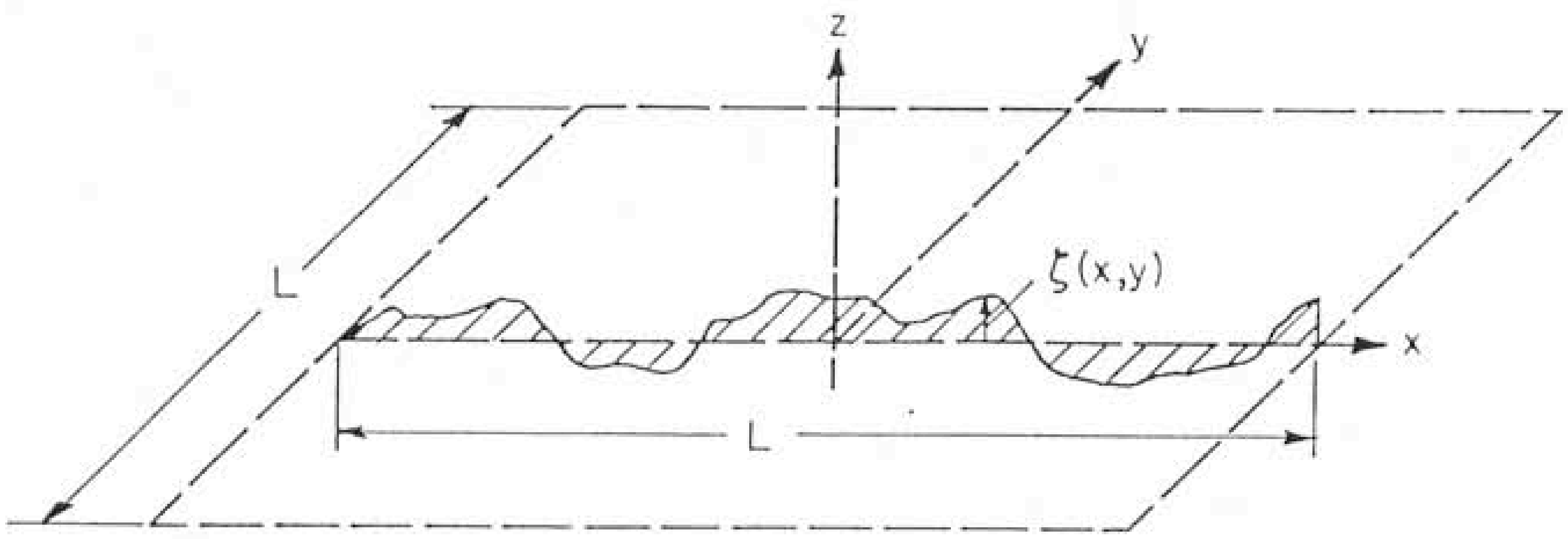

Fig. 1. Geometry of a slightly rough surface.

employed by extending the period to infinity and allowing the series to approach the Fourier integral. Rice developed this technique for statistical communication theory [*Rice,* 1944, 1945] and showed that the coefficients $P(m, n)$ become uncorrelated random variables in the limit as $L \to \infty$. Practically, it is sufficient that L be considerably larger than the correlation length of the surface height.

As a result of the fact that $\langle \zeta(x, y) \rangle = 0$ (statistical averages are denoted here by angle brackets), we can state that

$$\begin{aligned} &\langle P(m, n) \rangle = 0 \\ &\langle P(m, n)P(u, v) \rangle = 0 \quad \text{for} \quad u, v \neq -m, -n \\ &\langle P(m, n)P(u, v) \rangle = (\pi^2/L^2) W(p, q) \\ &\qquad\qquad \text{for} \quad u, v = -m, -n \end{aligned} \tag{2}$$

where $p = am = 2\pi m/L$ and $q = an = 2\pi n/L$. The function $W(p, q)$ defines the average height spectral density of the surface, and p, q are the radian wave numbers (or spatial frequencies) along the x and y directions, respectively.

The definition of the correlation coefficient of the height $R(\tau_x, \tau_y)$ for a homogenous surface illustrates the limiting process employed when the series becomes an integral

$$\begin{aligned} \langle \zeta(x, y)\zeta(x', y') \rangle &\equiv \sigma^2 R(\tau_x, \tau_y) \\ &= \sum_{m,n,u,v} \langle P(m, n)P(u, v) \rangle \\ &\quad \cdot \exp(iamx + iaux' + iany + iavy') \\ &= \sum_{m,n} \langle P(m, n)P(-m, -n) \rangle \\ &\quad \cdot \exp[iam(x - x') + ian(y - y')] \\ \underset{L\to\infty}{\longrightarrow} \frac{1}{4} &\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W(p, q) \exp(ip\tau_x + iq\tau_y)\, dp\, dq \end{aligned} \tag{3}$$

where $\tau_x = x - x'$ and $\tau_y = y - y'$ and σ^2 is the mean-square height of the surface.

Modal expansion for the guided wave fields. Let us first repeat the expression for a wave guided along the interface of a perfectly flat, highly conducting material of impedance Δ; this can be found in any standard text on electromagnetic theory such as *Jordan* [1950]. The wave is propagating in the x direction, as shown in Figure 2, with the H field perpendicular to the page. Then

$$E_x \simeq E_0\, \Delta \exp[ik_0(1 - \Delta^2)^{1/2} x - ik_0 \Delta z] \tag{4a}$$

$$E_y = 0 \tag{4b}$$

$$E_z \simeq E_0 \exp[ik_0(1 - \Delta^2)^{1/2} x - ik_0 \Delta z] \tag{4c}$$

where E_0 is the E field amplitude constant and the time dependence $\exp(-i\omega_0 t)$ is omitted.

When the surface is slightly rough, we express the total field above the interface as a perturbation of equations 4:

$$E_x = \Delta E(h, 0, z) + \sum_{m,n=-\infty}^{\infty} A_{mn} E(m + h, n, z) \tag{5a}$$

$$E_y = \sum_{m,n=-\infty}^{\infty} B_{mn} E(m + h, n, z) \tag{5b}$$

$$E_z = E(h, 0, z) + \sum_{m,n=-\infty}^{\infty} C_{mn} E(m + h, n, z) \tag{5c}$$

where

$$E(m + h, n, z) \equiv E_0 \exp[ia(m + h)x + iany + ib(m + h, n)z] \tag{6}$$

and

$$b(m + h, n) \equiv [k_0^2 - a^2(m + h)^2 - a^2 n^2]^{1/2} \tag{7}$$

The definition of b above is such that equations 5 satisfy the wave equation. The Cartesian components of the H field are not given here but are readily determined from Maxwell's equations.

In equations 5, the presence of the roughness manifests itself as the summation terms. In keeping

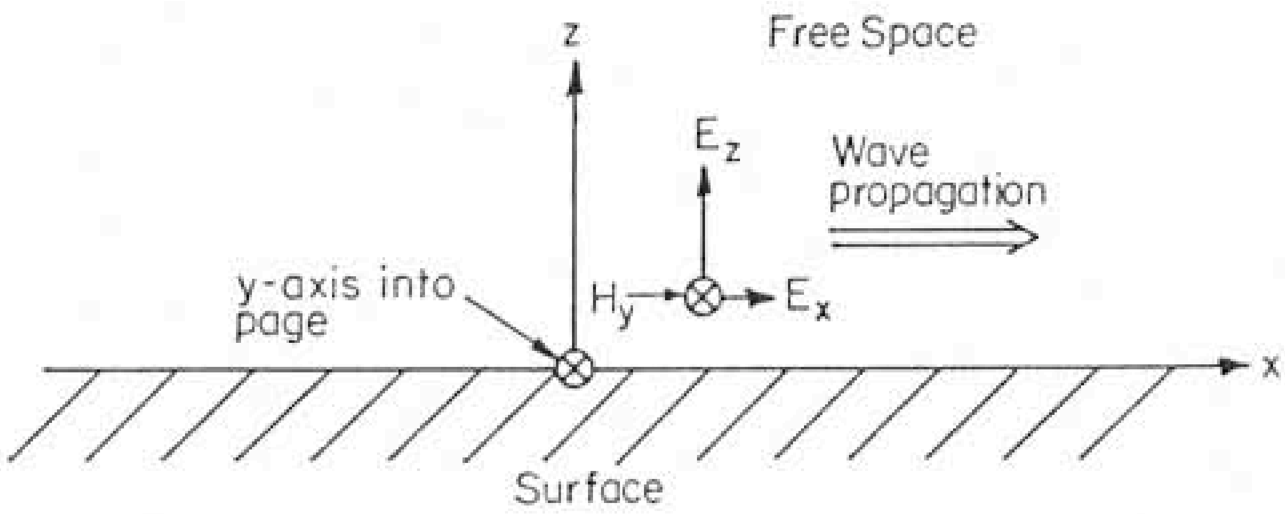

Fig. 2. Guided wave above a planar impedance boundary.

with the Rayleigh hypothesis (valid upon satisfaction of the first restriction above), we represent the field in terms of only upgoing waves. As the roughness disappears, A_{mn}, B_{mn}, and C_{mn} will vanish, and $b(h, 0) \rightarrow -k_0\Delta$; equations 5 then reduce identically to equations 4, valid above a smooth impedance boundary. Also, we take C_{00} to be identically zero. This choice is possible; what it really means is that the field is normalized such that at $x = y = 0$, the total z-directed E field amplitude is E_0.

Definition of the effective surface impedance. Physically, the guided-wave portions of the field appearing in equations 5 are all terms having the $E(h, 0, z)$ structure. These are

$$E_x^G = (\Delta + A_{00})E(h, 0, z) \quad (8a)$$

$$E_y^G = B_{00}E(h, 0, z) \quad (8b)$$

$$E_z^G = E(h, 0, z) \quad (8c)$$

By comparing equations 8 with equations 4, one is led to define the effective surface impedance as

$$\bar{\Delta} \equiv \Delta + A_{00} \quad (9a)$$

If the surface height is a random variable, the average effective surface impedance becomes

$$\bar{\Delta} \equiv \Delta + \langle A_{00} \rangle \quad (9b)$$

Physically, Equations 9 indicate that the average wave front and polarization tilt at the mean surface is $E_x^G/E_z^G = \bar{\Delta} = \Delta + A_{00}$, which is a common way of defining the surface impedance.

Leontovich boundary condition at the perturbed surface. The ultimate goal of the analysis is to determine A_{00}. To do this we shall substitute Equations 5 into the Leontovich boundary condition at the perturbed surface $z = \zeta(x, y)$. The Leontovich boundary condition is

$$\mathbf{E}_\zeta - (\hat{n}\cdot\mathbf{E}_\zeta)\hat{n} = 120\pi\,\Delta(\hat{n}\times\mathbf{H}_\zeta) \quad (9c)$$

where $\mathbf{E}_\zeta$ and $\mathbf{H}_\zeta$ are the total E- and H- field vectors above the perturbed surface ζ and $\hat{n}$ is the unit normal to the perturbed surface. It is defined as

$$\hat{n} = \frac{-\zeta_x\hat{x} - \zeta_y\hat{y} + \hat{z}}{(1 + \zeta_x^2 + \zeta_y^2)^{1/2}}$$

$$\simeq [-\zeta_x\hat{x} - \zeta_y\hat{y} + \hat{z}][1 - \tfrac{1}{2}(\zeta_x^2 + \zeta_y^2)] \quad (10a)$$

where $\hat{x}$, $\hat{y}$, and $\hat{z}$ are unit vectors along the coordinate axes. The approximation on the right side of (10) is possible because of the second restriction above, that the surface slopes are small; terms up to and including second order in ζ_x^2, ζ_y^2, and $(k_0\zeta)^2$ are to be retained throughout this analysis because it will turn out that A_{00} is of second order.

When (10a) is substituted into (9c) and the fields are represented by their Cartesian components, the x and y portions of the boundary condition, (9c), become

$$E_x - \zeta_x^2E_x - \zeta_x\zeta_yE_y + \zeta_xE_z = \frac{\Delta}{ik_0}\left\{-\zeta_y\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) - [1 - \tfrac{1}{2}(\zeta_x^2 + \zeta_y^2)]\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\right\} \quad (10b)$$

$$E_y - \zeta_y^2E_y - \zeta_x\zeta_yE_x + \zeta_yE_z = \frac{\Delta}{ik_0}\left\{-\zeta_x\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right) - [1 - \tfrac{1}{2}(\zeta_x^2 + \zeta_y^2)]\left(\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y}\right)\right\} \quad (10c)$$

In the above equations, Maxwell's equation has been used to express H in terms of E. No attempt is made in (10a) to order the fields; for example, E_z will turn out to be of zero order but E_y is of first order. Terms higher than second in the above equations will be dropped later.

Derivation of first-order coefficients of the perturbed fields. In deriving A_{00}, we follow the classic perturbation approach. The smallness parameters are the normalized surface height $(k_0\zeta)$, the surface slopes ζ_x and ζ_y, and the normalized impedance of the lower medium Δ. We shall retain terms up to second order in $(k_0\zeta)$, ζ_x, and ζ_y, but only up to first order in Δ (i.e., terms of the order of Δ^2 will not be saved). The reason for this is the fact that A_{mn}, B_{mn}, and C_{mn} are of *at least* first order in $(k_0\zeta)$, ζ_x, or ζ_y, whereas they are of the order of zero in Δ (e.g., if ζ, ζ_x, and ζ_y go to zero and the surface becomes smooth, the perturbed field coefficients disappear; if Δ goes to zero so that the surface becomes perfectly conducting, however, these coefficients do not vanish). Since the field coefficients are at least of first order in the surface parameters, we order them also as follows: $A_{mn} = A_{mn}^{(1)} + A_{mn}^{(2)} + \cdots$, with similar expressions for B_{mn} and C_{mn}. Thus $A_{mn}^{(1)}$ is the first-order contribution to A_{mn} and is associated with the first-order terms in (10).

We now substitute (5) into the boundary conditions (10); the exponentials $E(m+h, n, \zeta)$ involving the surface height are expanded; i.e.,

$$\exp[ib(m+h, n)\zeta] = 1 + ib(m+h, n)\zeta - \tfrac{1}{2}b^2(m+h, n)\zeta^2$$

Then these equations become:

$$\begin{aligned}
&\Delta[1 + ib(h, 0)\zeta - \tfrac{1}{2}b^2(h, 0)\zeta^2] \\
&\quad + \sum \{[A_{mn}^{(1)} + A_{mn}^{(2)}][1 - ib(m+h, n)\zeta - \tfrac{1}{2}b^2(m+h, n)\zeta^2]\} Ex(m, n) - \zeta_x^2\,\Delta + \zeta_x[1 + ib(h, 0)\zeta] \\
&\quad + \zeta_x \sum C_{mn}^{(1)} Ex(m, n) = \frac{\Delta}{ik_0}\{-\zeta_y \sum [B_{mn}^{(1)} i(m+h)a - A_{mn}^{(1)} ina] Ex(m, n) \\
&\quad - ib(h, 0)\,\Delta[1 + ib(h, 0)\zeta - \tfrac{1}{2}b^2(h, 0)\zeta^2] + iha[1 + ib(h, 0)\zeta - \tfrac{1}{2}b^2(h, 0)\zeta^2] \\
&\quad - \sum [((A_{mn}^{(1)} + A_{mn}^{(2)}) ib(m+h, n) - (C_{mn}^{(1)} + C_{mn}^{(2)}) i(m+h)a) \\
&\quad \cdot (1 + ib(m+h, n)\zeta)] Ex(m, n) + \tfrac{1}{2}(\zeta_x^2 + \zeta_y^2)[ib(h, 0)\,\Delta - iha]\} \qquad (11a)
\end{aligned}$$

$$\begin{aligned}
&\sum \{[B_{mn}^{(1)} + B_{mn}^{(2)}][1 + ib(m+h, n)\zeta]\} Ex(m, n) - \zeta_x\zeta_y\,\Delta + \zeta_y[1 + ib(h, 0)\zeta] \\
&\quad + \zeta_y \sum C_{mn}^{(1)} Ex(m, n) = \frac{\Delta}{ik_0}\{-\zeta_x \sum [A_{mn}^{(1)} ina - B_{mn}^{(1)} i(m+h)a] Ex(m, n) \\
&\quad - \sum [((B_{mn}^{(1)} + B_{mn}^{(2)}) ib(m+h, n) - (C_{mn}^{(1)} + C_{mn}^{(2)}) ina)(1 + ib(m+h, n)\zeta)] Ex(m, n)\} \qquad (11b)
\end{aligned}$$

where $Ex(m, n) = \exp(iamx + iany)$. The quantity $\exp(ihax)$ was common to every term and hence was dropped. Besides the above, we employ a third equation obtained from the divergence condition $\nabla \cdot \mathbf{E} = 0$:

$$\begin{aligned}
&ha(\Delta + A_{00}) + b(h, 0) \\
&\quad + \sum_{m,n\neq 0} [a(m+h)(A_{mn}^{(1)} + A_{mn}^{(2)}) \\
&\quad + an(B_{mn}^{(1)} + B_{mn}^{(2)}) \\
&\quad + b(m+h, n)(C_{mn}^{(1)} + C_{mn}^{(2)})] E(m, n, z) \qquad (11c)
\end{aligned}$$

We now collect terms of the order of zero in ζ, ζ_x, and ζ_y; from (11a) (there are no zero-order terms in (11b)) we obtain:

$$\Delta + (\Delta/ik_0)[ib(h, 0)\,\Delta - iha] = 0 \qquad (12a)$$

From (11c) the zero-order terms are equated to zero:

$$ha(\Delta + A_{00}) + b(h, 0) = 0 \qquad (12b)$$

The above equation shows that $b(h, 0)$ is small in terms of ha because Δ and A_{00} are small. Since $ha = [k_0^2 - b^2(h, 0)]^{1/2}$, we can use the binomial expansion to obtain

$$ha \simeq k_0\left(1 - \frac{1}{2}\frac{b^2(h, 0)}{k_0^2}\right) \qquad (12c)$$

Thus ha is very close to k_0, which is expected, since propagation of the guided wave is very nearly in the x direction.

Now let us collect and equate to zero all the first-order terms in ζ, ζ_x, ζ_y, $A_{mn}^{(1)}$, $B_{mn}^{(1)}$, and $C_{mn}^{(1)}$ found in (11). We use (12a) to eliminate terms from (11a). Finally, we employ (1) for the surface heights and slopes and equate to zero each term of the ensuing series multiplying $Ex(m, n)$ to give:

$$\left[1 + \Delta\frac{b(m+h, n)}{k_0}\right] A_{mn}^{(1)} - \Delta\frac{(m+h)a}{k_0} C_{mn}^{(1)} = -imaP(m, n) \qquad (13a)$$

$$\left[1 + \Delta\frac{b(m+h, n)}{k_0}\right] B_{mn}^{(1)} - \Delta\frac{na}{k_0} C_{mn}^{(1)} = -inaP(m, n) \qquad (13b)$$

$$(m+h)a\,A_{mn}^{(1)} + naB_{mn}^{(1)} + b(m+h, n)C_{mn}^{(1)} = 0 \qquad (13c)$$

We now use (13c) to eliminate $C_{mn}^{(1)}$ and solve for $A_{mn}^{(1)}$ and $B_{mn}^{(1)}$. When these are substituted into (13a) and (13b), we solve for the latter coefficients; the result is

$$A_{mn}^{(1)} = \frac{N_A}{D} P(m, n); \qquad B_{mn}^{(1)} = \frac{N_B}{D} P(m, n);$$

$$C_{mn}^{(1)} = \frac{-(m+h)aN_A - naN_B}{b(m+h, n)\, D} P(m, n) \qquad (14)$$

where

$$N_A = -ima\left[1 + \Delta\left(\frac{b(m+h, n)}{k_0} + \frac{n^2a^2}{k_0 b(m+h, n)}\right)\right] + ina\frac{\Delta(m+h)na^2}{k_0 b(m+h, n)} \qquad (15a)$$

$$N_B = -ina\left[1 + \Delta\left(\frac{b(m+h, n)}{k_0} + \frac{(m+h)^2a^2}{k_0 b(m+h, n)}\right)\right] + ima\frac{\Delta(m+h)na^2}{k_0 b(m+h, n)} \qquad (15b)$$

$$D = \left[1 + \Delta\left(\frac{b(m+h, n)}{k_0} + \frac{(m+h)^2a^2}{k_0 b(m+h, n)}\right)\right]\left[1 + \Delta\left(\frac{b(m+h, n)}{k_0} + \frac{n^2a^2}{k_0 b(m+h, b)}\right)\right] - \frac{\Delta^2(m+h)^2n^2a^4}{k_0^2 b^2(m+h, n)} \qquad (15c)$$

The above equations show that the scattered or perturbed field coefficients can be separated into two factors. The first involves N_A, N_B, and D and depends only on the electrical properties of the material beneath the surface (as contained in Δ) and in the directions of propagation of the scattered plane wave (as contained in am and an). The dependence on the shape of the surface is contained entirely in $P(m, n)$. Furthermore, the m, nth Fourier component of the surface gives rise to scatter in a unique direction determined by $a(m + h)$, an; this direction is precisely the Bragg direction [*Barrick and Peake*, 1967, 1968], and the m, nth component of the surface is acting as a diffraction grating.

The object of the present analysis is the derivation of A_{00}. But since $P(0, 0)$ was taken to be zero (i.e., the mean plane is the $z = 0$ plane), (14) shows that $A_{00}^{(1)}$ is identically zero. Hence, A_{00} must be of second order in ζ, ζ_x, and ζ_y.

Derivation of the second-order coefficient $A_{00}^{(2)}$. $A_{00}^{(2)}$ can be determined entirely from (11a). By equating the second-order terms and again using (12a) to eliminate terms, we obtain

$$\sum\left[\left(1 + \Delta\frac{b(m+h, n)}{k_0}\right)A_{mn}^{(2)} - \Delta\frac{(m+h)a}{k_0}C_{mn}^{(2)}\right]Ex(m, n) = \frac{\Delta}{2}(\zeta_x^2 - \zeta_y^2) + ik_0\Delta\zeta\zeta_x$$
$$- \sum\left\{\left[ib(m+h, n)\zeta\left(1 + \Delta\frac{b(m+h, n)}{k_0}\right) - \Delta\frac{na}{k_0}\zeta_y\right]A_{mn}^{(1)} + \Delta\frac{(m+h)a}{k_0}\zeta_y B_{mn}^{(1)}\right.$$
$$\left. + \left[-ib(m+h, n)\zeta\,\Delta\frac{(m+h)a}{k_0} + \zeta_z\right]C_{mn}^{(1)}\right\}Ex(m, n) \qquad (16)$$

The right side of the above equation will involve double summation sets when we introduce the series for ζ, ζ_x, and ζ_y. In order to employ the orthogonality relationship of the eigenfunctions $Ex(m, n)$ and set individual terms of the series on the left side equal to zero, it is convenient to rearrange the double summation. The examples below illustrate the procedure.

$$\zeta_x^2 = \sum_{\alpha,\beta}\sum_{\gamma,\zeta}(i\alpha a)(i\gamma a)P(\alpha, \beta)P(\gamma, \zeta)\cdot Ex(\alpha+\gamma, \beta+\zeta)$$
$$\equiv \sum_{m,n}\sum_{k,l} ia(m-k)(iak)P(k, l)\cdot P(m-k, n-l)Ex(m, n) \qquad (17a)$$

$$\sum_{m,n} ib(m+h, n)\zeta A_{mn}^{(1)} = \sum_{\alpha,\beta}\sum_{\gamma,\zeta} ib(\alpha+h, \beta)\cdot P(\gamma, \zeta)A_{\alpha\beta}^{(1)}Ex(\alpha+\gamma, \beta+\zeta)$$
$$\equiv \sum_{m,n}\sum_{k,l} ib(k+h, l)A_{kl}^{(1)}\cdot P(m-k, n-l)Ex(m, n) \qquad (17b)$$

The right side of (16) is then expressed so that $Ex(m, n)$ shows explicitly, and then the individual terms of the series in m, n multiplying $Ex(m, n)$ are equated to zero. Since we are interested in $A_{00}^{(2)}$, only the $m = n = 0$ term is to be studied here.

The left side of (16) reduces to

$$\left(1 + \Delta\frac{b(h, 0)}{k_0}\right)A_{00}^{(2)} - \Delta\frac{ha}{k_0}C_{00}^{(2)}$$

This result can be simplified further because C_{00} was defined earlier to be zero to all orders, according to our normalization. Furthermore, the term $\Delta\, b(h, 0)/k_0$ is of at least second order compared to unity, as can be seen from (12*b*). Hence the left side of (16) reduces to $A_{00}^{(2)}$, and we obtain

$$A_{00}^{(2)} = \sum_{k,l} \Bigg\{ [-ib(k+h, l) A_{kl}^{(1)} + iakC_{kl}^{(1)}] + \Delta\left[k_0 ak + \frac{a^2k^2 - a^2l^2}{2} \right] P(k, l) - \Delta\Bigg[\left(\frac{ia^2l^2}{k_0} + \frac{ib^2(k+h, l)}{k_0} \right) A_{kl}^{(1)} - \frac{ia^2(k+h)l}{k_0} B_{kl}^{(1)} - \frac{ib(k+h, l)a(k+h)}{k_0} C_{kl}^{(1)} \Bigg] \Bigg\} P(-k, -l) \tag{18}$$

We now employ (14) and (15) for the first-order coefficients. Algebraic simplification is possible, but we spare the reader the details here. Terms are discarded of order Δ^2 and higher. We then obtain the following expression for $A_{00}^{(2)}$:

$$A_{00}^{(2)} \simeq \sum_{k,l} \left\{ \frac{k_0 a^2 k^2}{b(k+h, l)\ D(k, l)} + \Delta\left[\frac{a^2k^2 + a^2l^2 - k_0 ak}{D(k, l)} + \left(k_0 ak + \frac{a^2k^2 - a^2l^2}{2} \right) \right] \right\} |P(k, l)|^2 \tag{19}$$

where $D(k, l)$ can also be simplified by discarding higher terms in Δ to give

$$D(k, l) \simeq 1 + \frac{k_0}{b(k+h, l)} \Delta\left[\frac{b^2(k+h, l)}{k_0^2} + 1 \right] \tag{20}$$

Thus we have obtained a result for $A_{00}^{(2)}$ that is a summation over the Fourier components of the surface height. When $\Delta \to 0$, the above equation reduces to that of *Rice* [1951]. The second term in (20) thus provides a correction and estimate of the effect of finite conductivity; for a Δ that approaches unity, this correction term alone is obviously inadequate and another technique must be sought.

Effective surface impedance. From (9), we have the expression for the effective impedance of a periodic surface with Fourier components $P(m, n)$:

$$\bar{\Delta} = \Delta + \sum_{k,l} \left\{ \frac{k_0 a^2 k^2}{b(k+h, l)\ D(k, l)} + \Delta\left[\frac{a^2k^2 + a^2l^2 - k_0 ak}{D(k, l)} + \left(k_0 ak + \frac{a^2k^2 - a^2l^2}{2} \right) \right] \right\} |P(k, l)|^2 \tag{21}$$

If the surface is a pure sinusoid along the direction of propagation, for example, then $k = \pm 1$ and $l = 0$; hence the above series reduces to two terms. If the sinusoid is oriented along the y axis so that propagation is along the crestlines, then $k = 0$ and $l = \pm 1$. In this case, $A_{00}^{(2)} = 0$, $\bar{\Delta} = \Delta$, and the guided wave is not influenced by the presence of the corrugations.

If we are dealing with a random surface and desire a statistical average of $\bar{\Delta}$, as expressed in (9*b*), the summation can be reduced to an integral yielding

$$\bar{\Delta} = \Delta + \frac{1}{4} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(p, q) W(p, q)\, dp\, dq \tag{22}$$

where

$$F(p, q) = \frac{p^2 + b'\ \Delta(p^2 + q^2 - k_0 p)}{b' + \Delta(b'^2 + 1)} + \Delta\left(\frac{p^2 - q^2}{2} + k_0 p \right) \tag{23a}$$

$$b' = \frac{1}{k_0} [k_0^2 - (p + k_0)^2 - q^2]^{1/2} \tag{23b}$$

We have used the approximation above that $ha \simeq k_0$, which is valid so long as $\bar{\Delta}$ is small compared to unity as seen from (12*c*) and (12*b*).

As a check on (22), if we permit Δ to approach zero in the integrand, we can obtain a form that agrees with *Feinberg*'s [1944] result (after proper normalizations), i.e.,

$$\bar{\Delta} = \Delta + \frac{1}{4} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{p^2 W(p, q)\, dp\, dq}{b'} \tag{24}$$

Thus we have in (21) and (22) expressions valid to order Δ for the effective surface impedance at grazing of imperfectly conducting, rough surfaces.

INTERPRETATION OF THE INTERACTION PROCESS

Power flow near the surface. Following equations 8, we defined the $m = n = 0$ terms in equations (5) for the fields as the guided wave; all the other terms we will refer to as the scattered field. The scattered field owes its existence strictly to the presence of roughness. The strength of the modes in the scattered field, as represented by A_{mn}, B_{mn}, and C_{mn}, are proportional to the Fourier coefficients of the roughness height $P(m, n)$, as seen from (14). These scattered field terms consist of both upgoing, propagating plane-wave modes and also evanescent, non propagating modes. These two modes are distinguished by the exp $[ib(m + h, n)z]$ factor: when m and n are such that b (defined by (7)) is real, the mode is propagating; when m and n are sufficiently large that b is the square root of a negative number, this factor is $\exp[-|b(m+h,n)|z]$ for $z > 0$ and the modes are evanescent.

An understanding of the interaction process is facilitated by examining the power flow near the surface. We employ the complex Poynting vector $\mathbf{S} = \frac{1}{2}(\mathbf{E} \times \mathbf{H}^*)$, where an average over time is implied. Furthermore, we intend to average over one spatial period, which for convenience we take to be the square of side L centered at $x, y = 0$, as shown in Figure 1; the spatial average is denoted by $\tilde{\mathbf{S}}$. This merely means that we are interested in the net power flow over at least one complete roughness cycle (or hill) of the surface, rather than over a portion of a hill or cycle. It is not the same as a statistical average, however; statistical averaging can be performed on S in *addition* to the spatial averaging, if appropriate.

The derivations of the $x-$, $y-$, and $z-$ components of $\tilde{\mathbf{S}}$ is straightforward. The spatial averaging reduces the double summation over m, n, k, l to a single set over m, n. Finally, we set $z = 0$ because we are interested in the power flow just above the mean surface. The results, correct to the second order, are:

$$\tilde{S}_x = \frac{E_0^2}{4k_0(120\pi)}\left[k_0 + \sum_{m,n}(am + k_0)(|A_{mn}^{(1)}|^2 + |B_{mn}^{(1)}|^2 + |C_{mn}^{(1)}|^2)\right] = \tilde{S}_x^G + \tilde{S}_x^s \tag{25a}$$

$$\tilde{S}_y = \frac{E_0^2}{4k_0(120\pi)}\sum_{m,n} an(|A_{mn}^{(1)}|^2 + |B_{mn}^{(1)}|^2 + |C_{mn}^{(1)}|^2) = \tilde{S}_y^s \tag{25b}$$

$$\tilde{S}_z = \frac{E_0^2}{4k_0(120\pi)}\left[-k_0(\Delta + A_{00}^{(2)}) + \sum_{m,n} b(m+h,n)(|A_{mn}^{(1)}|^2 + |B_{mn}^{(1)}|^2 + |C_{mn}^{(1)}|^2)\right] = \tilde{S}_z^G + \tilde{S}_z^s \tag{25c}$$

where $\tilde{S}^G$ refers to the power flow in the guided-wave portion of the fields, represented by the first term in (25*a*) and (25*c*) above; $\tilde{S}^s$ denotes the power flow contained in the summations. Hence the $\tilde{S}^s$ terms include the power in the scattered fields. The scattered power density in these three directions could have been defined without our resorting to the Poynting vector technique; it is merely the square of the amplitudes in each of the three orthogonal polarization states for each mode, multiplied by the direction cosine of that mode along the three principal directions. These direction cosines are $(am + k_0)$, an, and $b(m + h, n)$; hence, we have a check on our interpretation.

It is evident from equations 25 that $\tilde{S}_x$ and $\tilde{S}_y$ are pure real, but $\tilde{S}_z$, the power flow upward (away from the surface), is complex in general. $\tilde{S}_z^s$ is imaginary when $b(m+h,n)$ is imaginary; the latter, we have seen, occurs when m and n are sufficiently large that the mode is evanescent. An imaginary Poynting vector, of course, means that the power is reactive and hence is not dissipated but rather stored near the surface. This is consistent with the nature of fields near the teeth on guiding structures, where induction and electrostatic fields are created around closely spaced grooves, such that am and an are large in terms of k_0, the radio wave number.

Alternate definition of effective surface impedance. Earlier we defined the effective surface impedance from inspection of equations 5 and 8. It may not have been altogether obvious that the entire contribution due to roughness comes with $A_{00}^{(2)}$, the zero-order mode of E_z. A more formal definition can be made in terms of the power flow. *Norton* [1937*b*], *Wise* [1937], and *Jordan* [1950] all have shown that the ratio of the Poynting vector propagating downward into the surface to the Poynting vector propagating in the forward direction is equal to the normalized surface impedance Δ. This is true whether one is dealing with a guided wave, as shown by Jordan, or with the energy radiated from a dipole, as discussed by Norton and Wise. Hence, the ratios of these Poynting vectors should provide another meaningful definition of the effective surface impedence at grazing.

Extending this to a roughened surface, one would logically define $\bar{\Delta}$ as the ratio of the downgoing Poynting vector to the Poynting vector to the unperturbed guided wave in the forward direction. From (25), this is $\bar{\Delta} = -\tilde{S}_z^G/\tilde{S}_x^G$, where it is clear from (25*c*) that the downgoing power flow is represented in the first term with the minus sign. Thus we arrive at identically the same expression for $\bar{\Delta}$ as before, i.e., equation 9, where $\bar{\Delta} = \Delta + A_{00}^{(2)}$.

Thus we see that the average downward flow of power, $\tilde{S}_z^G$, consists of two portions. The portion represented by Δ enters the surface (as though it were perfectly smooth) and is converted to heat due to ohmic losses, as represented by the finite conductivity of the medium. The second downward-going portion, proportional $A_{00}^{(2)}$, is due to roughness and is made up primarily of the upgoing scattered power density, as represented by $\tilde{S}_z^s$. It consists of energy removed from the guided-wave mode by scatter and hence is dissipated. There is a final small portion of $\tilde{S}_z^G$ contained in $A_{00}^{(2)}$ that is *not* balanced by $\tilde{S}_z^s$; this is the perturbation in the power that enters the surface, owing to the roughness. It can be found by subtracting $\tilde{S}_z^s$ from that portion of $-\tilde{S}_z^G$ containing $A_{00}^{(2)}$; it is of the order of Δ.

If we had neglected the last small component of power mentioned above, an alternate method of defining $\bar{\Delta}$ would have involved computing only the first-order scattered fields (both propagating and evanescent); no second-order derivation of $A_{00}^{(2)}$ would have been necessary. From conservation of power, one could have argued that the upgoing scattered power, i.e., $\tilde{S}_z^s$, must be balanced by a downgoing component equal to it. Thus for a perfectly conducting surface, one would define

$$\bar{\Delta} = \frac{[E_0^2/4k_0(120\pi)]}{[E_0^2/4k_0(120\pi)]} \sum_{m,n} b(m + h, n) (|A_{mn}^{(1)}|^2 + |B_{mn}^{(1)}|^2 + |C_{mn}^{(1)}|^2)$$

Using equation 14 for $A_{mn}^{(1)}$, $B_{mn}^{(1)}$, and $C_{mn}^{(1)}$ with $\Delta = 0$, one arrives at

$$\bar{\Delta} = \sum_{m,n} \frac{k_0 a^2 m^2}{b(m + h, n)} |P(m, n)|^2 \qquad (26)$$

which is identically equal to (21) with $\Delta = 0$. Hence, the various definitions are equivalent and provide a coherent interpretation of the interaction mechanism. Equation 26, when averaged statistically, reduces to the integral in equation 24.

Interpretation of the resistive and reactive components of $\bar{\Delta}$. We now examine the first-order contributions to $\bar{\Delta}$ (the effective surface impedance), which result from the roughness. The first-order effect is expressed in (26) for a deterministic periodic surface, or in the integral in (24) for a random surface where Δ is taken as zero in the integrand. The integral is pure, real, and positive when b' is real and is imaginary and negative when b' is imaginary; the former case represents a resistive contribution to $\bar{\Delta}$, whereas the latter is a reactive contribution.

Physically, the real or resistive portion of $\bar{\Delta}$ comes from roughness components whose normalized spatial wave numbers ($am/k_0 = p/k_0$, $an/k_0 = q/k_0$) lie within the unit circle centered at $-1, 0$, as shown in Figure 3. This is possible only if there are components of the roughness height spectral density $W(p, q)$ with

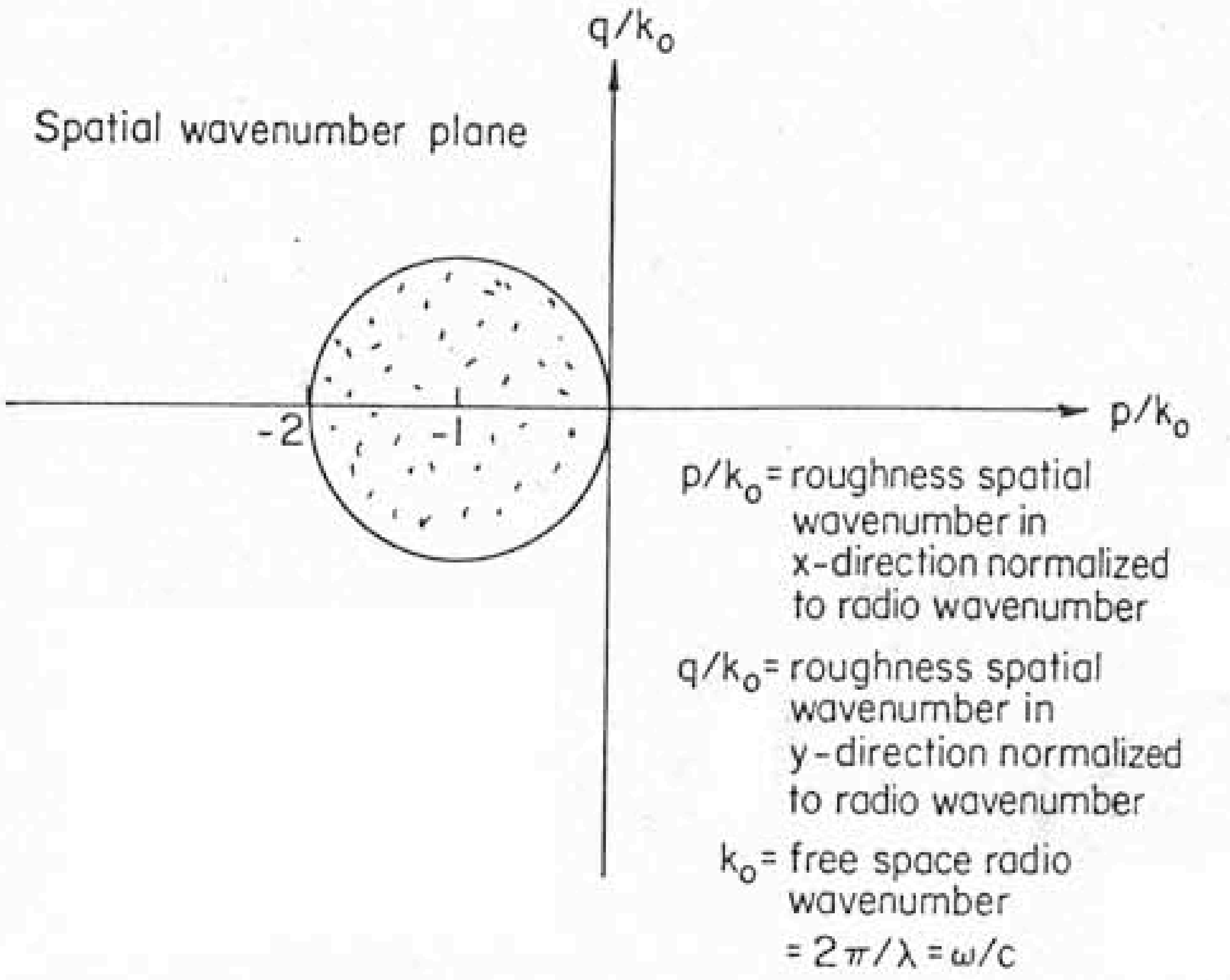

Fig. 3. Effect of various regions of spatial roughness spectrum on effective surface impedance; propagation direction (x axis) corresponds to p/k_0 axis. Roughness spectrum within unit circle contributes to the effective surface resistance R_Δ; the remainder of the spectrum contributes to the effective surface reactance X_Δ.

spatial frequencies less than $2k_0$. Physically, this means that only roughness waves longer than one-half the radio wavelength can contribute to the resistive portion. But it is precisely these roughness waves whose wave numbers lie within the unit circle that produce propagating scattered fields by the Bragg mechanism [*Barrick and Peake*, 1967, 1968]. As seen from (25), these longer roughness waves are responsible for the removal of energy from the guided wave and scatter into all directions in the upper hemisphere; this active energy removal produces the resistive term of the surface impedance.

On the other hand, those components of the roughness spectrum with periods shorter than $\lambda/2$ (lying outside the unit circle) yield a reactive component of the surface impedance. In addition, this negative reactance always represents an inductance (due to our time convention $e^{i\omega t}$). Roughness waves of these shorter lengths do not produce propagating scattered modes but, rather, evanescent modes. As seen from (25), the power associated with these shorter roughness components is reactive also; hence, the reactive portion of the impedance is directly related to the reactive power density at the surface.

Let us examine a slightly rough perfectly conducting surface with spatial periods all less than l_0 and assume that we begin to increase the frequency of a wave guided across the surface. The first effect we notice is an increase in the effective inductance of the surface, with no resistive contribution at all as long as $k_0 < \pi/l_0$. When the frequency is such that $k_0 > \pi/l_0$ and when some roughness components lie within the unit circle, then we will suddenly notice an increase in the resistive portion. Hence, at lower radio frequencies, a given rough surface will appear inductive, but at higher frequencies the resistive term will eventually increase until it is as large as the inductive term. This conclusion assumes of course that the height still remains small compared to the radio wavelength.

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Theory of HF and VHF propagation across the rough sea, 2, Application to HF and VHF propagation above the sea

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This paper deals with an estimation of the effect of sea state on HF and VHF ground-wave propagation loss across the ocean. For this estimation, an expression is employed that was derived in part 1; it gives the effective surface impedance in terms of the spatial height spectrum of the surface. Two empirical models are employed for the height spectrum of the ocean, the directional Neumann-Pierson model and the isotropic Phillips wind-wave model. These effective surface impedances accounting for sea state are then inserted into a standard ESSA computer program giving the basic transmission loss above a spherical earth. The dependence on sea state is plotted in curves which show the excess loss in decibels over a smooth sea. The propagation loss to points at various heights above the sea is also calculated. Normal sea state variations are seen to be negligible below about 2 MHz but produce a maximum excess loss at about 10 to 15 MHz.

USE OF $\bar{\Delta}$ FOR RADIATION ABOVE THE EARTH'S SURFACE

In Part 1, we derived an expression for $\bar{\Delta}$, the effective impedance of a slightly rough, finitely conducting planar surface at grazing incidence for vertical polarization. To do this we assumed a sourceless guided wave in our description of the fields above the interface. There is the question How does one employ this result in the analysis of radiation from a source above a rough spherical surface such as the sea?

Norton [1937] has shown that the field radiated from a dipole at the surface has the appearance locally of a guided wave, even though the asymptotic expansion of the total radiated field at large distances contains no guided wave term. More specifically, the total radiated field near a point on the surface can be expanded in a series about that point; its first terms are the same as those of the guided wave. The horizontal distance ρ from this point within which the fields appear to be guided is related to the 'numerical distance' of Norton such that $\rho \sim \lambda/\pi\,|\Delta|^2$ [see *Wait*, 1964]. Hence, for highly conducting surfaces such that $|\Delta| \ll 1$, the region about a point on the surface within which the field appears guided is many wavelengths in extent. *Wait* [1964] shows that this is true for spherical surfaces also in his expansions for the height-gain function. In addition, one can demonstrate that even the sum of the incident and reflected rays appears guided near the surface within a neighborhood of radius ρ about a surface point.

Therefore, since the fields far from a source appear guided in nature within a region of radius ρ (defined above) about a surface point, one is justified in treating the fields near an irregular surface of period L as though they are guided, so long as $L < \rho$. As an example, for the sea at 10 MHz, $\rho \simeq 70$ km. Hence the expansion of the fields into guided-wave modes in part 1 is valid since the water waves of practical interest are less than 1 km in length.

Thus the value of effective surface impedance derived in part 1, equations 21 and 22, can now be used in any of the standard analyses of radiation above the earth at near-grazing angles. The slightly rough surface is conceptually replaced by a smooth, flat surface at $z = 0$ with effective impedance $\bar{\Delta}$. (The same procedure is used, for example, with rectangularly corrugated surfaces, where a $\bar{\Delta}$ is derived by *Elliott* [1954] using waves of a guided nature and static-field approximations for the fields between the narrowly spaced teeth.) *Wait* [1957, 1964] shows that problems involving radiation above both a planar and a spherical earth can be formulated in terms of the normalized surface impedance, and in fact, he shows Elliott's value of $\bar{\Delta}$ as applicable when the surface is corrugated.

We intend in this paper to employ $\bar{\Delta}$, derived from equation 22 for the sea, in a standard ESSA program provided by *Berry and Chrisman* [1966] for propagation above a spherical surface. In this manner, we shall study the effect of sea state on propagation loss above the ocean at HF and VHF.

WIND-WAVE SPECTRUM MODELS FOR THE OCEAN

At HF, the longer, higher gravity waves will obviously play an important role in the interaction process. Such ocean waves owe their existence to winds blowing above the surface. However, a precise, quantitative relationship between the roughness heights and the winds is complicated by several factors. First, it takes several hours for waves to build up to their full strength under the influence of the wind; for example, it takes 20 hours for the sea to build to its full or 'saturated' height under 15-knot winds. In most instances, the winds will not stay constant either in magnitude or direction for this long a period, and hence the sea is rarely 'saturated' at the longer wavelengths. Second, the homogeneity of the wave spectrum aroused in a given region depends on the spatial area over which the winds are blowing; this area is called the 'fetch' and may be as small as 100 km in length. Third, part of the roughness is due to winds or storms located far away in space and/or time from the region of interest; this is called 'swell.' It is usually quite directional and 'narrowband,' and hence its spectrum $W(p, q)$ appears as 'spikes' in the p-q plane.

With these complications, the only exact way of obtaining the true ocean wave height spectrum in a given area is to measure it. For purposes of obtaining a rough estimate of the effect on propagation, however, we will employ two semi-empirical models with a minimum of parameters. The models, developed by oceanographers over the past 15 years, neglect the effect of limited fetch and swell and they assume the winds have been blowing sufficiently long that the sea is fully developed. Empirical evidence indicates that the wave height spectrum follows a law close to κ^{-4} in the saturated region ($\kappa^2 = p^2 + q^2$) and that it falls off rapidly to zero at the lower end when $\kappa < g/U^2$, where U is the wind speed (m/sec) and g is the acceleration of gravity.

We will examine two models. The first is a Neumann-Pierson spectrum that is modified to give some preference to waves moving in the direction α from the x axis. The assumed directionality is cosine squared, as proposed by *Kinsman* [1965], and the assumed form is

$$W(p, q) = \frac{C(p \cos \alpha + q \sin \alpha)^2}{g^{5/2}(p^2 + q^2)^{13/4}} \cdot \exp \{-2g/[U^2(p^2 + q^2)^{1/2}]\} \qquad (1)$$

where C is a constant empirically estimated to be 3.05 m^2/sec^5. This spectrum has an inverse 9/2-power over-all dependence on κ in the saturated region (i.e., for κ large); the exponential function is an artificial factor producing a smooth but rapid lower-end cutoff. The spectrum is assumed to be nonzero only in the half space into which the wind is blowing. The mean-square height of ocean waves obtained from this model is $\sigma^2 = \frac{3}{2} C(\pi/2)^{3/2} \cdot (U/2g)^5$.

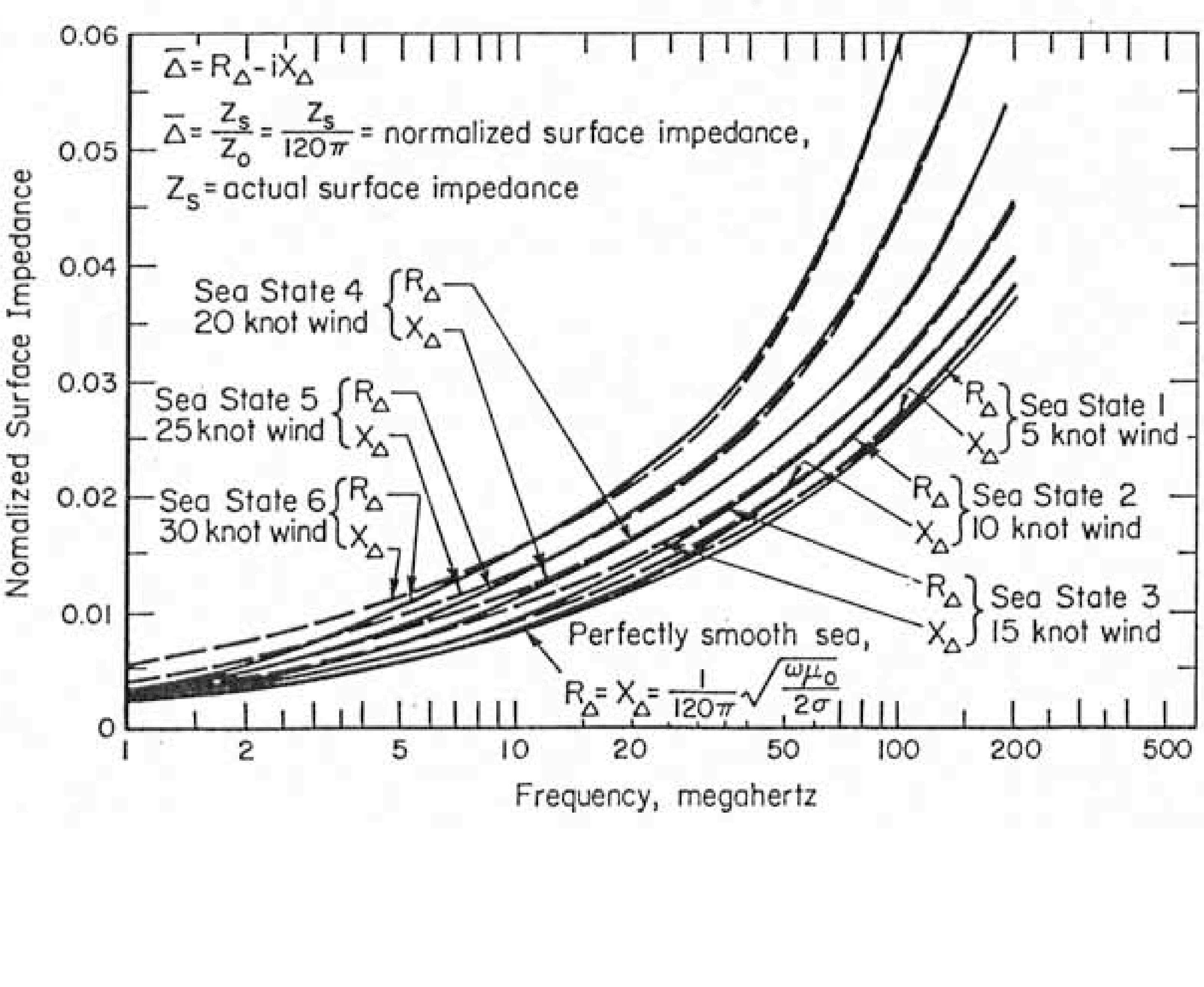

Fig. 1. Effective surface impedance $\bar{\Delta}$ versus wind speed and frequency. Neumann-Pierson ocean-wave spectrum and propagation in up-wind-downwind direction.

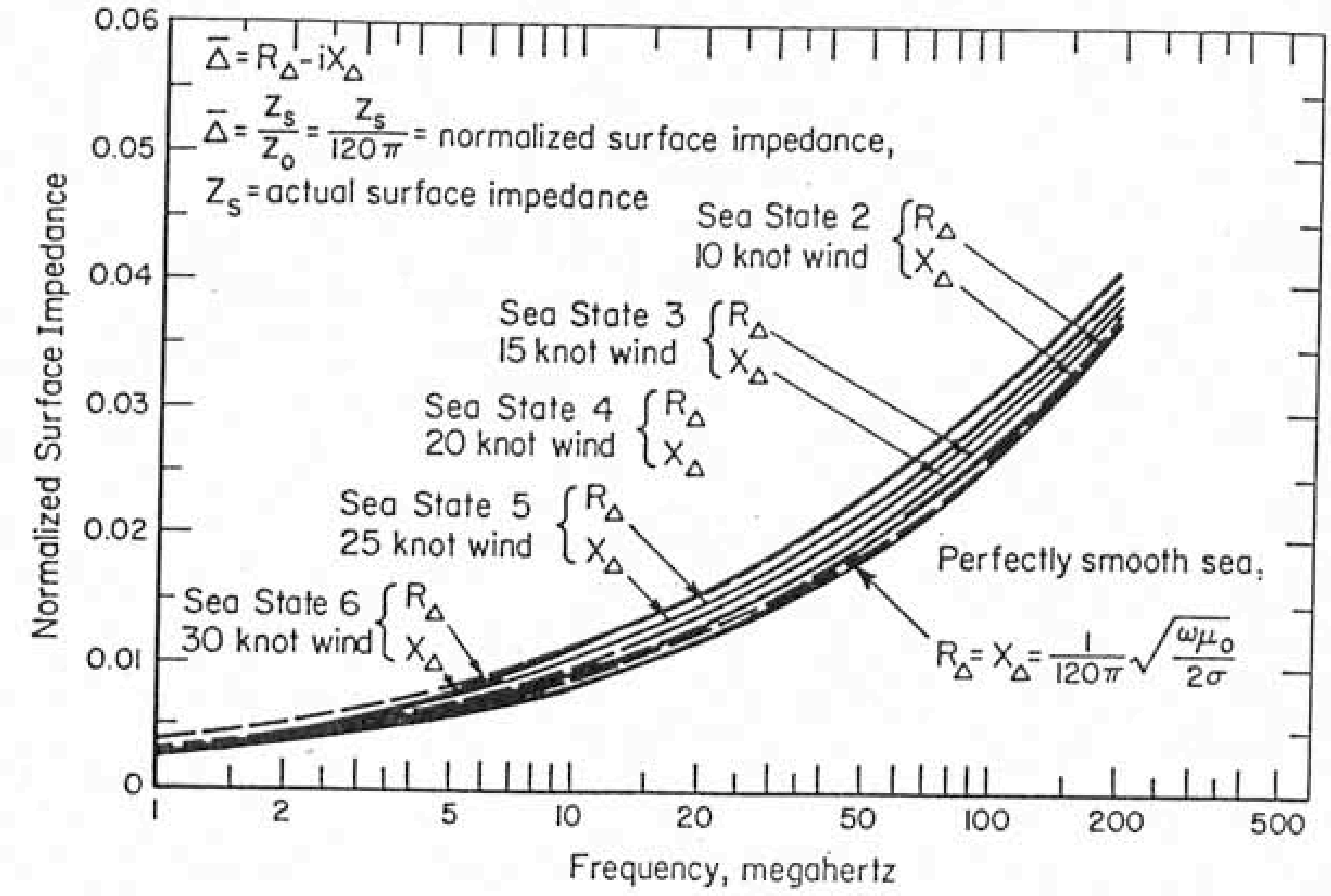

Fig. 2. Effective surface impedance $\bar{\Delta}$ versus wind speed and frequency. Neumann-Pierson ocean-wave spectrum and propagation in crosswind direction.

The second model we will consider is attributed to *Phillips* [1966]. Basing their conclusions on ocean-wave data gathered more recently, *Phillips* [1966] and *Munk and Nierenberg* [1969] have presented evidence to show that κ^{-4} dependence is a closer fit than $\kappa^{-9/2}$. In addition, measurements show that the lower-end cutoff is much more pronounced than the exponential factor in the Neumann-Pierson model. Thirdly, slope measurements by *Cox and Munk* [1954] show that the spectrum is closer to being isotropic than the cosine-squared directionality assumed in (1). Hence they suggest the following spectrum:

$$W(p, q) = 4B/\pi(p^2 + q^2)^2 \qquad (2)$$

where $B \simeq 0.005$. The spectrum is identically zero for $\kappa < g/U^2$ and also in the half space from which the wind is blowing. The mean-square height of the waves from this model is $\sigma^2 = \frac{1}{2} BU^4/g^2$.

It must be mentioned that very little oceanographic data have been gathered that support any given directional behavior of a general model such as these. For this reason we examine both models here to observe the effect of directionality on the effective impedance of the surface.

EFFECTIVE SURFACE IMPEDANCE FOR TWO WIND-WAVE MODELS

The averaged effective impedance $\bar{\Delta}$ of a random surface such as the sea is expressed in (22) of part 1 in terms of $W(p, q)$, the average wave height spatial spectrum of the surface. We shall now employ the

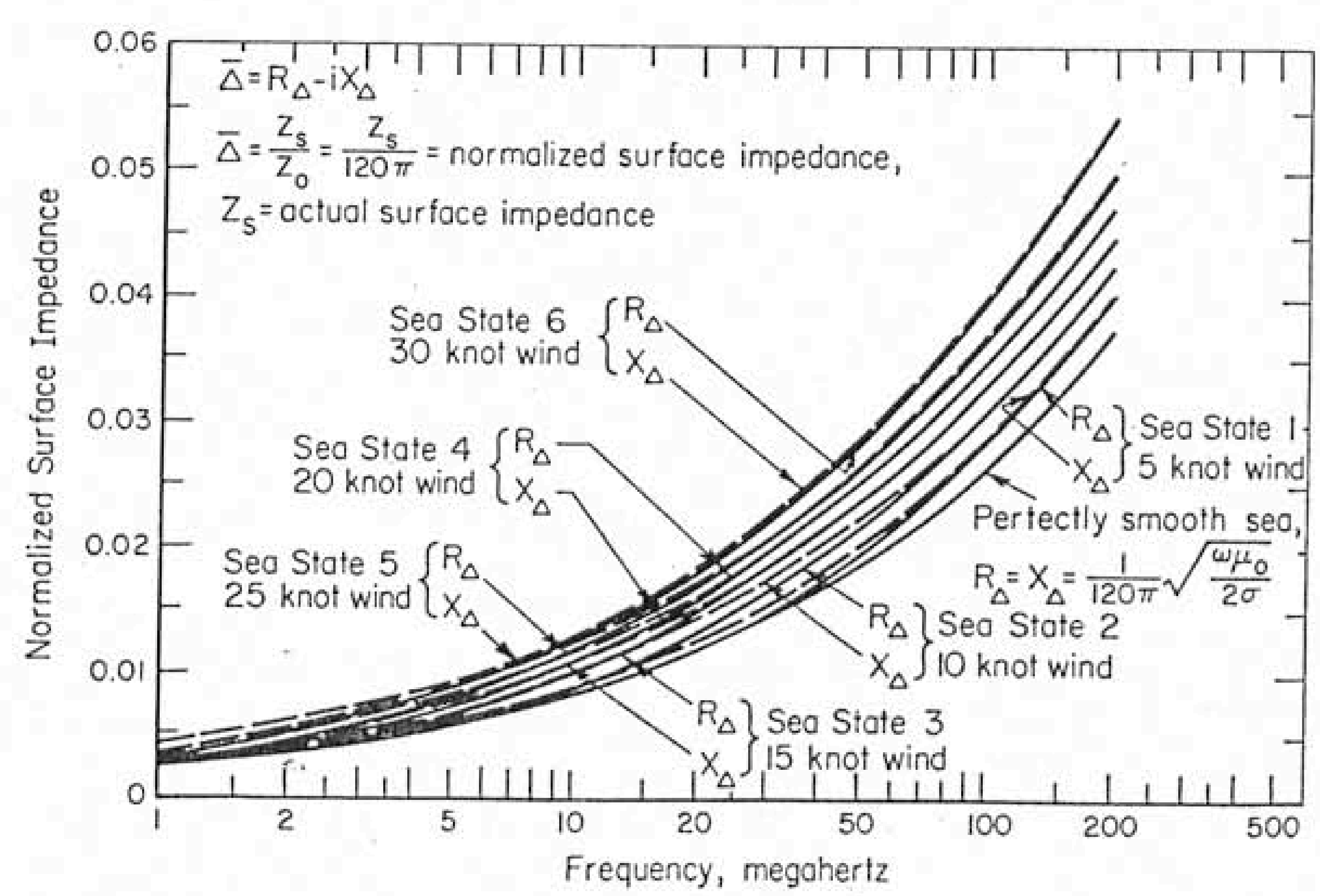

Fig. 3. Effective surface impedance $\bar{\Delta}$ versus wind speed and frequency. Phillips' isotropic ocean-wave system.

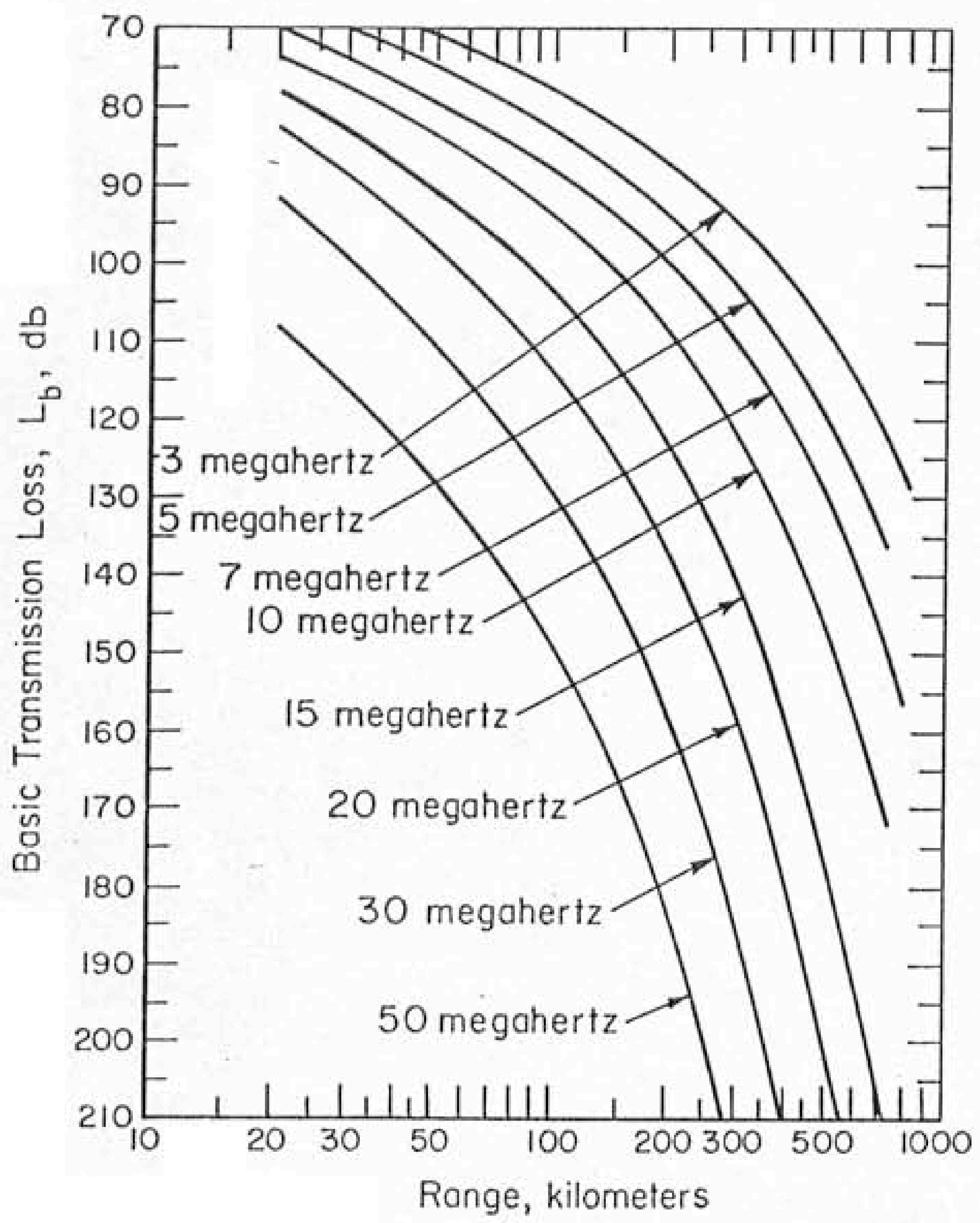

Fig. 4. Basic transmission loss across the ocean between points at the surface of *smooth* spherical earth. Conductivity is 4 mhos/m and an effective earth radius factor of $\frac{4}{3}$ is assumed.

two semi-empirical wind-wave models for $W(p, q)$ described in the preceding section. For the directional Neumann-Pierson model, we examine two dominant wind (or wave) directions: first, along the direction of propagation ($\alpha = 0$), which we term the 'upwind-downwind' direction and, second, perpendicular to the direction of propagation ($\alpha = \pi/2$), which we term the 'crosswind' direction.

In both cases, we evaluate the integral numerically. We also divide by two and assume that the spectrum exists symmetrically over all space, instead of over only the forward wind half. This is necessary because at a given instant the sea profile will appear 'frozen' to a radio wave, and it will not be possible from this profile to tell whether the waves are moving forward or backward.

Figures 1 and 2 show the effective surface impedance $\bar{\Delta}$ for the Neumann-Pierson model of (1) for the upwind-downwind and crosswind directions, respectively. The broken lines in the figures represent the imaginary part of the impedance and the continuous lines represent the resistive part. The lowest line is the limit of surface impedance as the roughness vanishes, i.e., $\bar{\Delta} = \Delta$, where Δ is the wave impedance of sea water. We assume $\sigma = 4$ mho/m and $\epsilon_r = 80$, which are median values for the Atlantic Ocean at about 40° latitude. Figure 3 shows the calculated impedance for the Phillips semi-isotropic spectrum of (2), where the same constants are assumed for the sea water.

As seen in Figures 1 and 2, the most severe changes in surface impedance occur for propagation in the upwind-downwind direction (i.e., across the corrugations); this is, of course, expected from the presence of the p^2 factor in (22) and (24).

The figures also indicate that the first noticeable change in the surface impedance at the lower frequences appears as an inductive increase in the reactance. This means that nearly all the ocean waves are shorter than the radio wavelength; hence, most of the spectrum lies outside the unit circle shown in Figure 3 of part 1. As the sea becomes rougher (or as frequency increases), the ocean develops waves longer than the radio wavelength and the resistive portion becomes about equal to the reactive part.

Even at lower frequencies, however, the reactive portion is at most about twice the resistive portion. The resistive portion in this case is made up entirely of the real part of Δ, the impedance of ocean water. In fact, throughout the HF and VHF regions, the increase in impedance due to roughness is of the same order as the impedance of ocean water itself. Hence, the surface cannot be taken to be a perfect conductor. If the water were a better conductor, the effect of roughness would be more apparent; if the water conductivity were poorer (as with fresh water),

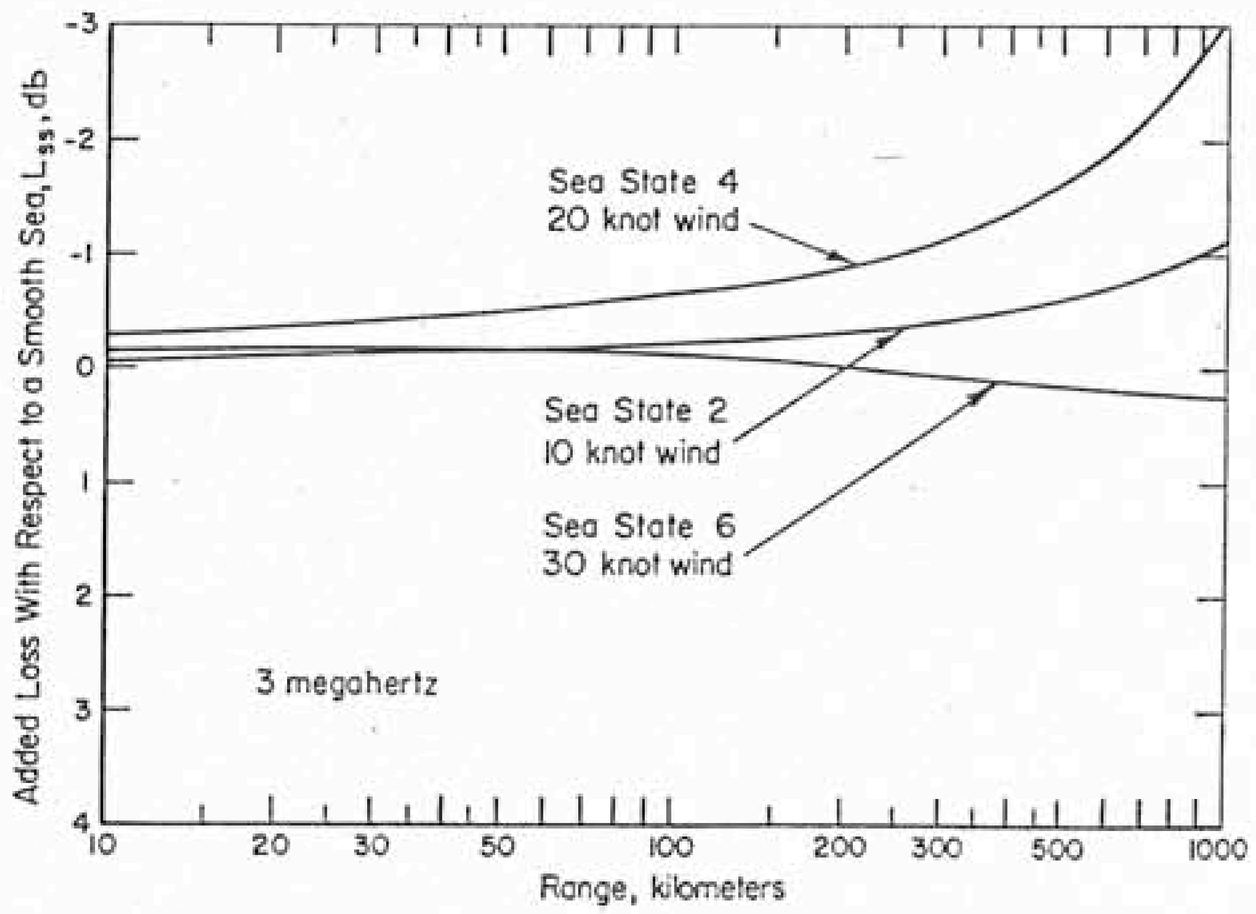

Fig. 5. Added transmission loss due to sea state at 3 MHz. Antennas are located just above surface. Phillips' isotropic ocean-wave spectrum.

Δ would be higher and the percentage increase due to roughness would be less. As it is, roughness on the sea is seen to produce an increase in $\bar{\Delta}$ that is by no means negligible.

INCREASED PROPAGATION LOSS DUE TO SEA STATE

We shall now examine the increased propagation loss due to roughness for a vertically polarized ground wave propagating above the sea surface near grazing. To do this, we will employ the values of effective surface impedance for the rough ocean in a standard formulation for propagation above a spherical earth. In particular, we use a Fortran 4 program developed by *Berry and Chrisman* [1966] of ESSA that has been modified to calculate basic transmission loss. As a compromise, the values of effective surface impedance obtained from the Phillips semi-isotropic wind-wave spectrum will be selected here (Figure 3), since they lie between the values of the Neumann-Pierson model for the upwind-downwind and crosswind directions. Again, a value of water conductivity of 4 mho/m is selected because it is typical of the Atlantic Ocean at mid-latitudes.

As a reference, we show in Figure 4 the basic transmission loss between two points just above the surface of a perfectly smooth spherical sea ($\sigma = 4$ mho/m, earth radius factor accounting for refractivity is 4/3) from the program. This basic transmission loss, widely publicized by *Norton* [1953, 1957], is defined as $L_b = 10 \log_{10} (P_{ti}/P_{ri})$ (decibels), where P_{ti} is the power transmitted by an isotropic radiator and P_{ri} is the power received by

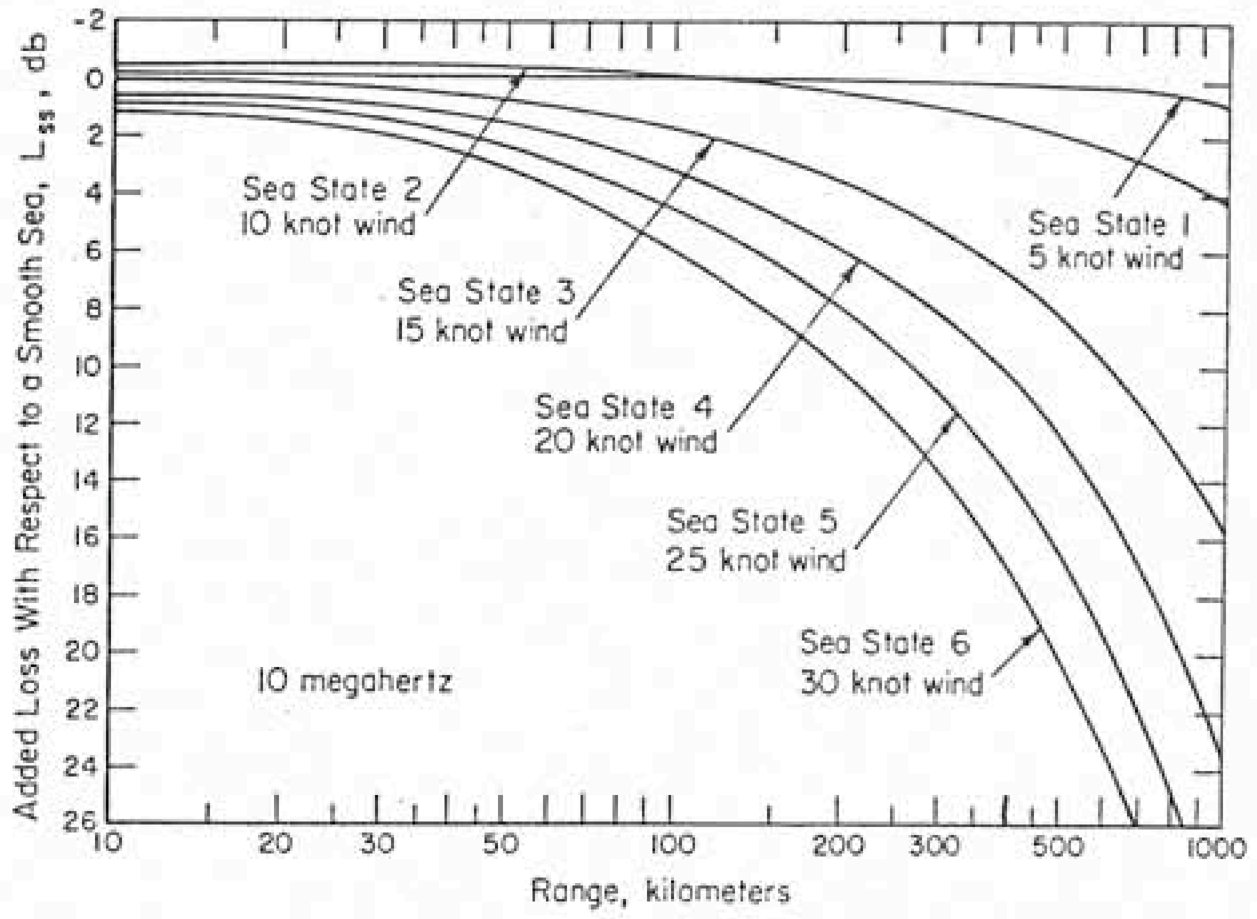

Fig. 7. Same as Figure 5, except added transmission loss due to sea state at 10 MHz.

an isotropic radiator. In a simple communication problem, one must merely subtract out the free-space antenna gains (in db) in order to determine the over-all power loss. As a check, the basic transmission loss between two points in free space separated by a distance d is $10 \log_{10} (4\pi d/\lambda)^2$, where λ is the wavelength.

In order to show explicitly the increased loss due to sea surface roughness, we subtract the basic transmission loss above a rough sea from that value shown in Figure 4 above a smooth sea; we call this the excess transmission loss due to sea state L_{SS}, in db. The transmitter and receiver points are both located on the surface. Figures 5 through 9 show this loss at different frequencies as a function of range (or distance) between transmitter and receiver.

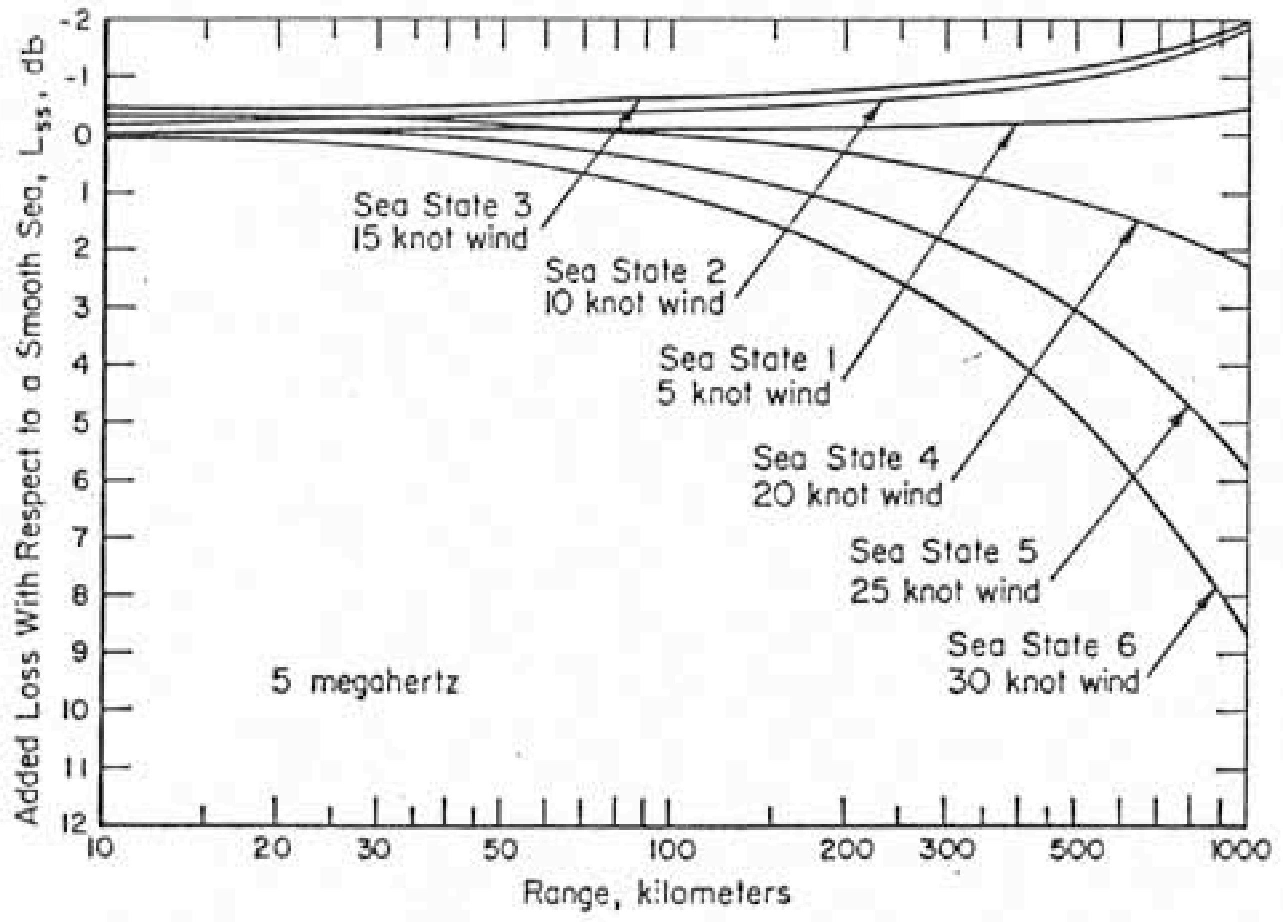

Fig. 6. Same as Figure 5, except added transmission loss due to sea state at 5 MHz.

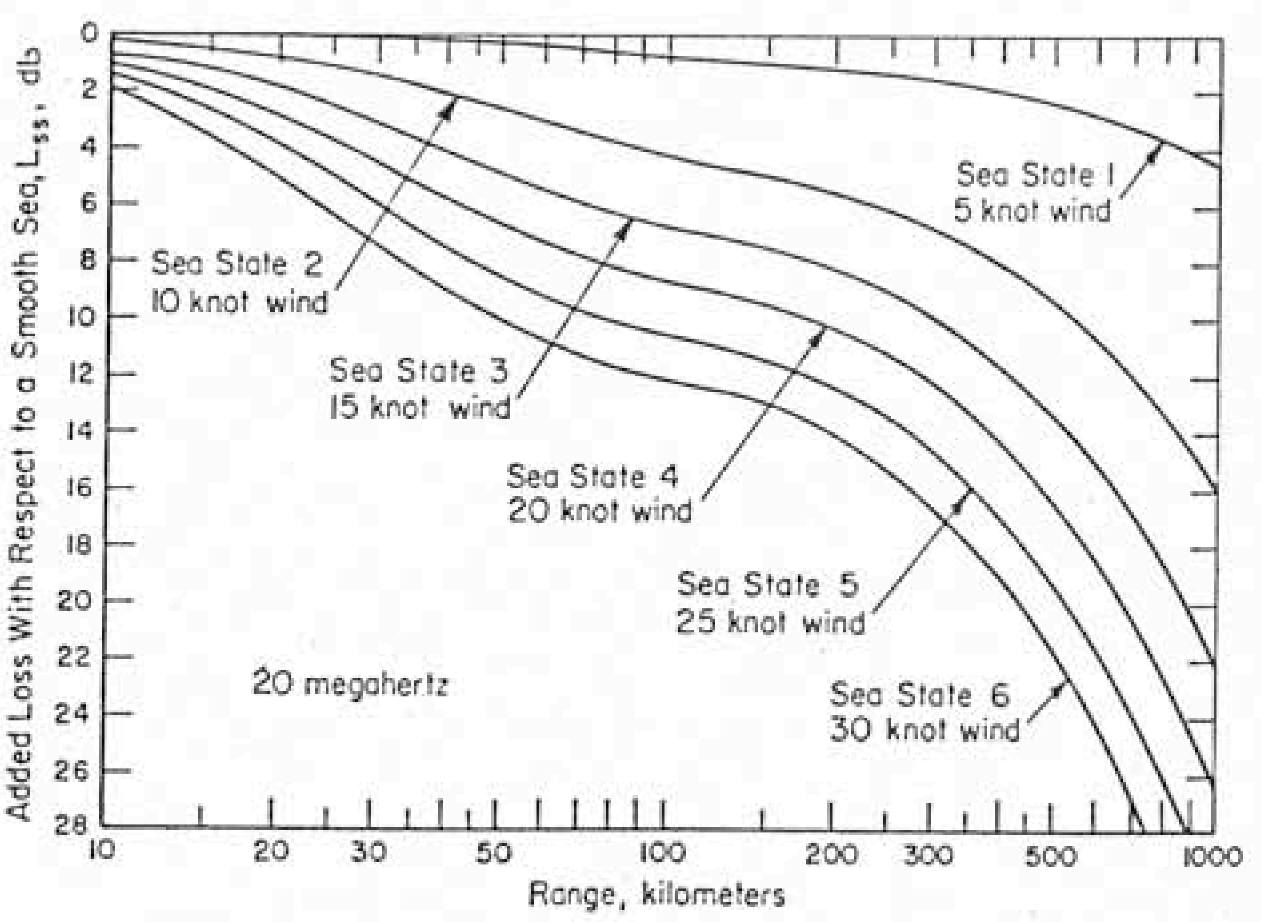

Fig. 8. Same as Figure 5, except added transmission loss due to sea state at 20 MHz.

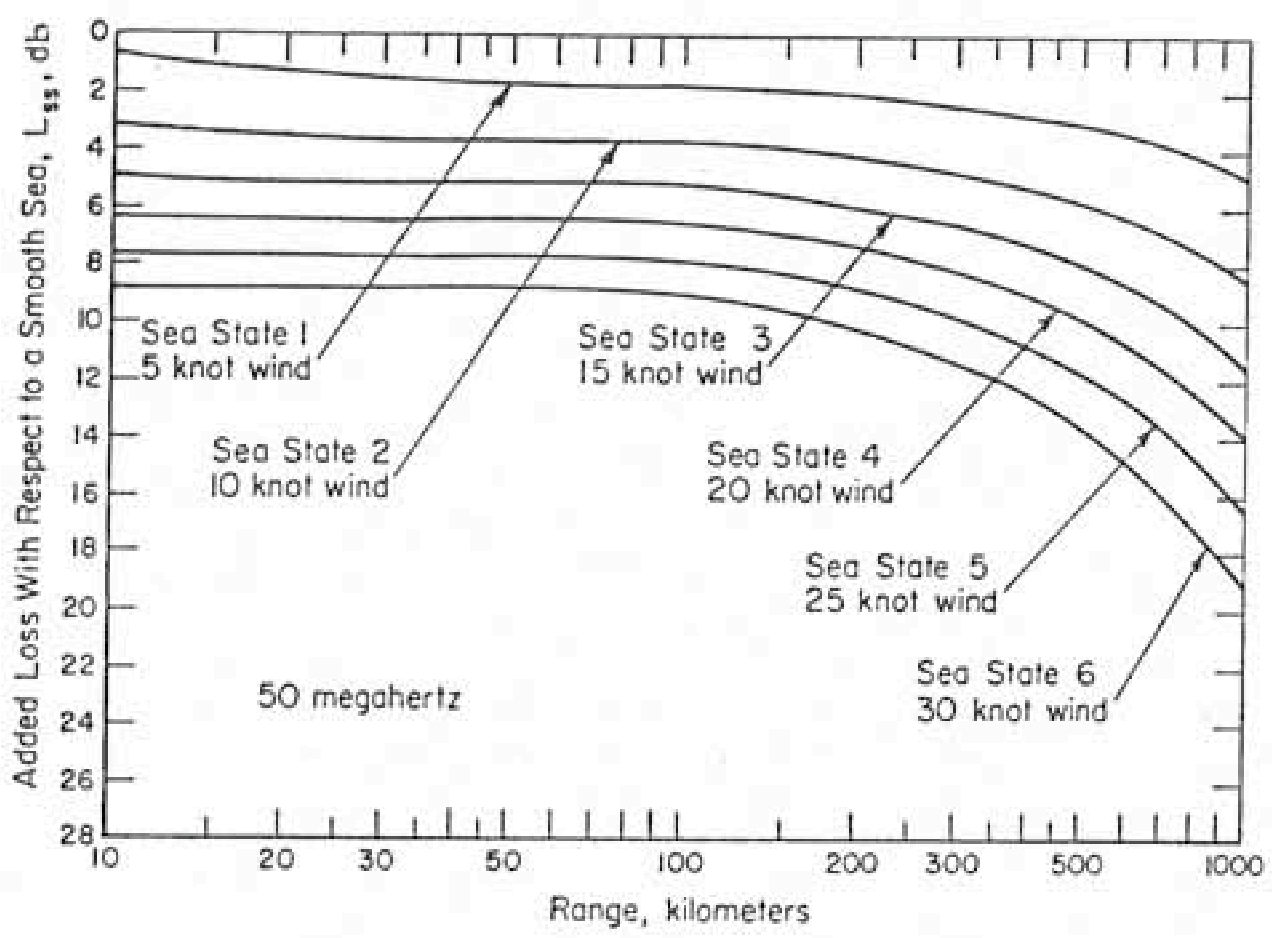

Fig. 9. Same as Figure 5, except added transmission loss due to sea state at 50 MHz.

In Figure 10 we show an example of the basic transmission loss at 10 MHz from a point on the surface to various points above the earth. (By reciprocity these numbers are also valid for the transmitter-receiver locations interchanged.) The first number at each height-range grid point is the loss when the sea is perfectly smooth, and the second grid point represents the loss when the sea is fully excited by a 25-knot wind. The Phillips wind-wave spectrum is employed here also. The 'height-gain' effect is clearly evident as one moves upward at a given range; first, there is a drop in signal of 2 to 3 db and then a monotonic increase as one moves out of the surface-wave region into the lit, or space-wave, region. More curves and grids of this type can be found in *Barrick* [1970].

SUMMARY AND CONCLUSIONS

The principal conclusion of this paper is that sea state can markedly affect the ground-wave loss for propagation across the ocean at HF and VHF. This increased loss will be greatest for rougher seas, higher frequencies, and longer ranges. For example, below 2 MHz, sea state effects are generally negligible, but at 15 MHz the increase in one-way loss can be as much as 15 db at 100 nmi.

In part 1 we developed the theory for the interaction of a guided wave with a slightly rough, finitely conducting surface. A convenient way of accounting for the presence of roughness is found to be the normalized effective surface impedance. This is shown to consist of two terms; one term is due to the smooth surface alone and the other depends on the height spectrum of the roughness. The effective surface impedance is simply related to the power flow in the scattered and evanescent modes produced by the radio wave interaction with the ocean waves.

In this paper the expression for the effective surface impedance is employed to calculate the ground-wave loss due to sea state. To obtain a rough quantitative estimate of this loss, we examined both directional and semi-isotropic versions of the Neumann-Pierson and Phillips wind-wave spatial spectrums. Finally, we used the values of surface impedance calculated for the Phillips spectrum to predict the excess transmission loss due to sea state.

Figures 5 through 9 for the excess loss show some significant effects. First, a negative loss is observed at lower frequencies; this actually indicates an increase in signal. This effect is expected and occurs where the increase in impedance is purely reactive, namely where the ocean waves present have lengths small in terms of the radio wavelength. *Wait* [1964] examines and discusses this 'trapping' which occurs when the reactance is greater than the resistance. The trapping effect here is not pronounced, however, because the resistance due to finite conductivity is never that much less than the total reactance. It is doubtful that such a small signal increase could ever be measured.

It is also interesting to note that sea state loss appears to be the greatest at about 10 to 15 MHz and seems to decrease above this frequency. This is apparently due to the increase in the normal wave impedance of ocean water with frequency, which becomes a significant percentage of the total effective impedance. Thus, a saturation appears to occur in the upper HF region.

The restrictions required for the validity of the theory (discussed in part 1) are easily satisfied by the sea up to the mid-VHF region. Conductivity is sufficiently great for ocean water that the Leontovich boundary condition is applicable. If water waves have height-to-length ratios that exceed 0.14 for any appreciable time, breaking occurs; hence the slope restriction is valid. The requirement that the height be small in terms of wavelength can be translated to a restriction on wind speed for a particular frequency by use of the mean-square heights for the two wind-wave models given after (1) and (2). By requiring that $(k_0\sigma)^2 \leq 0.2$, for example, the Phillips spectrum shows that the impedance and loss calculations presented here are valid at 50 MHz for seas driven by winds of up to 25 knots. Above this Rayleigh height limit, it is at present unclear whether extrapolation

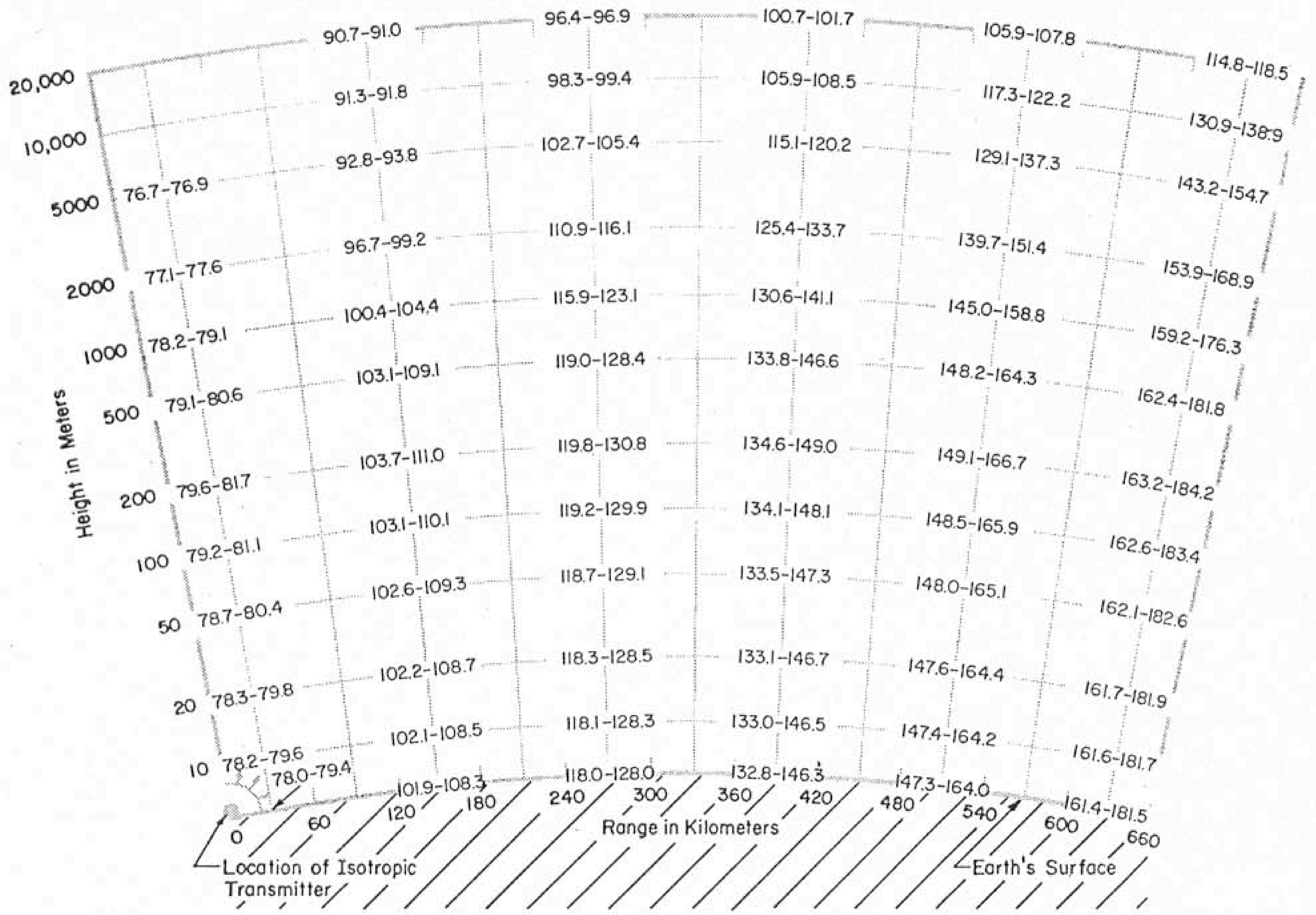

Fig. 10. Basic transmission loss (in decibels) to points at various heights and ranges above the ocean at 10 MHz. The first number is for perfectly smooth sea, and the second is for sea state 5 (25-knot wind); Phillips' ocean-wave spectrum is used.

of the curves can represent a valid and correct solution.

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